Elasticity and Plasticity

By now, you probably know about the concept of elasticity. In layman terms, it means that some substances get back to their original shape after being stretched. You have played with a slingshot. Haven't you? That is an elastic material. Let us get into the concepts of elasticity and plasticity and learn more about these two properties of matter.

Elasticity and Plasticity

Elasticity is the property of a body to recover its original configuration (shape and size) when you remove the deforming forces. Plastic bodies do not show a tendency to recover to their original configuration when you remove the deforming forces. Plasticity is the property of a body to lose its property of elasticity and acquire a permanent deformation on the removal of deforming force.

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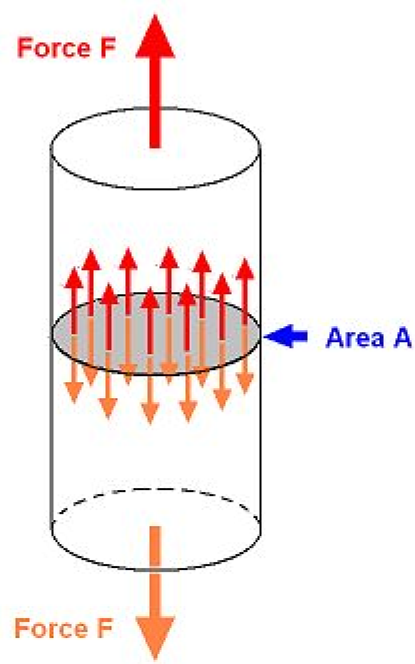
- Elastic Moduli

- Hooke's Law and Stress-strain Curve

Stress

The restoring force (F) per unit area (A) is called stress. The unit of stress in S.I system is N/m^2 and in C.G.S-$dyne/cm^2$. The dimension of stress = $[M^1L^{-1}T^{-2}]$. Stress is given by,

$$\text{Stress} = F/A$$



Types of Stress

Stress could be of the following types:

- Normal stress:- Normal stress has the restoring force acting at right angles to the surface.
    - Compressional stress:- This stress produces a decrease in length per volume of the body.
    - Tensile stress:- This stress results in an increase in length per volume of the body.
- Tangential stress:- Stress is said to be tangential if it acts in a direction parallel to the surface.

Strain

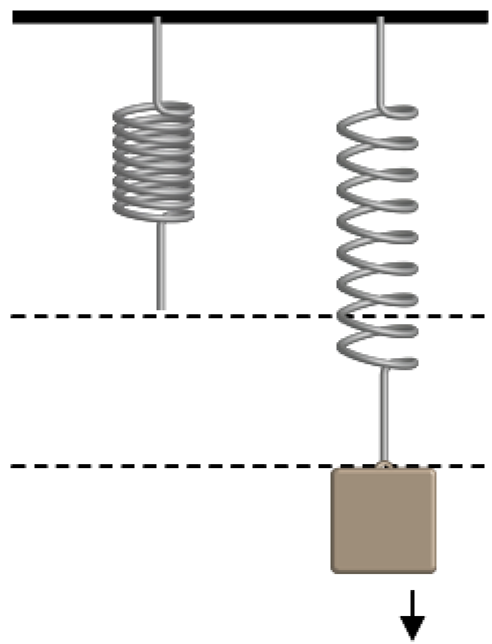
The strain is the relative change in configuration due to the application of deforming forces. It has no unit or dimensions. The strain could be of the following types:

- Longitudinal Strain: It is the ratio between the change in length (l) to its original length (L). Longitudinal strain = l/L
- Lateral Strain: The lateral strain is the ratio between the change in diameter to its original diameter when the cylinder is

subjected to a force along its axis. Lateral strain = change in diameter /original diameter

- Volumetric Strain: It is the ratio between the change in volume (v) to its original volume (V). Volume strain = v/V.

Hooke's Law



It states that within elastic limits, stress is proportional to strain. Within elastic limits, tension is proportional to the extension. So, Stress ∝ Strain or F/A∝l/L. Therefore, we have for:

- Stretching: Stress = Y×strain or $Y=F_{stretch}L/A(l)$

- Shear: Stress = η×strain or $\eta=F_{shear}L/A(l)$
- Volume Elasticity: Stress = B×strain or B = – P/(v/V)

Proportionality Coefficients

- The coefficient of elasticity: It is basically the ratio between stress and strain.
- Young's modulus of elasticity (Y): It is the ratio between normal stress to the longitudinal strain. Y = normal stress/longitudinal strain = (F/A)/(l/L) = (Mg×L)/(πr^2×L)
- Bulk modulus of elasticity (B): It is the ratio between normal stress to the volumetric strain. B = normal stress/volumetric strain = (F/A)/(v/V) = pV/v

Video on Elasticity

Other Important Terms

Compressibility

The compressibility of a material is the reciprocal of its bulk modulus of elasticity. Compressibility = 1/B.

Workdone in Stretching

- Workdone, $W = \frac{1}{2} \times (\text{stress}) \times (\text{strain}) \times (\text{volume}) = \frac{1}{2} Y(\text{strain})^2 \times \text{volume} = \frac{1}{2} [(\text{stress})^2/Y] \times \text{volume}$
- Potential energy stored, $U = W = \frac{1}{2} \times (\text{stress}) \times (\text{strain}) \times (\text{volume})$
- Potential energy stored per unit volume, $U = \frac{1}{2} \times (\text{stress}) \times (\text{strain})$

Workdone During Extension (Energy Density)

$W = \frac{1}{2} F \times l = \frac{1}{2}$ tension $\times$extension

Elastomer

Elastometer produces a large strain with a small stress.

Elastic Fatigue

The phenomenon by virtue of which a substance exhibits a delay in recovering its original configuration if it had been subjected to a stress for a longer time, is called elastic fatigue.

Poisson's Ratio (σ)

Poisson's ratio of the material of a wire is the ratio between lateral strains per unit stress to the longitudinal strain per unit stress. $\sigma =$

lateral strain/longitudinal strain = $\beta/\alpha = (\Delta D/D)/(\Delta L/L)$. Values of σ lie between -1 and 0.5.

Relations Among Elastic Constants

- $B = Y/[3(1-2\sigma)]$
- $\eta = Y/[2(1+\sigma)]$
- $9/Y = 3/\eta + 1/B$
- $\sigma = [3B-2\eta]/[6B+2\eta]$

Solved Examples For You

Q: Which of the following is/are true about deformation of a material?

A. Deformation capacity of the plastic hinge and resilience of the connections are essential for good plastic behaviour.
B. Deformation capacity equations considering yield stress and gradient of the moment.
C. Different materials have different deformation capacity.
D. All of the above.

Solution: D) In a well-designed steel frame structure, inelastic deformation under severe seismic loading is confirmed in beam plastic hinges located near the beam-to-column connections. Thus, deformation capacity of the plastic hinge and resilience of the connections are essential for good plastic behaviour at the hinge is strongly influenced by the difference of material properties.

Generally, the material properties are specified in terms of yield stress and/or ultimate strength. However, the characteristics of the materials are not defined by only these properties. Thus, the characteristics of various materials aren't reflected in present building codes, particularly on deformation capacity classification.

Applications of Elastic Behaviour of Materials

Have you seen a stretched slingshot? You surely must have played with it, haven't you? What happens when you release it? This is an important concept of elasticity and the Elastic behaviour of substances. It finds various applications in our day to day lives. Let us look at this concept in a greater detail.

Elastic Behaviour of Materials

A slingshot deforms when you stretch it. However, it regains its original shape when you stop applying a force. But let us say that you take a thin steel rod and try to bend. You manage to bend it a little and then stop applying force. Does the rod regain its original shape? No, it doesn't. This difference in the behaviour of the material is based on their elastic and plastic nature.

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Why does this happen?

The rubber strip of the slingshot has high elasticity. Elasticity is the ability of a body to resist any permanent change to it when stress is applied. When stress application ceases, the body regains its original shape and size.

Different materials show different elastic behaviour. The study of the elastic behaviour of a material is of much importance. Almost every engineering design requires knowledge of the elastic behaviour of materials.

Applications of this Concept

In the construction of various structures like bridges, columns, pillars, beams, etc. Knowledge of the strength of the materials used in the construction is of prime importance.

For example: While constructing a bridge, a load of traffic that it can withstand should be adequately measured beforehand. Or while constructing a crane used to lift loads, it is kept in mind that the extension of the rope does not exceed the elastic limit of rope. To overcome the problem of bending under force the elastic behaviour of the material used must be considered primarily.

To study the elastic behaviour of materials let us consider a beam of length l, breadth b and depth d supported at the ends and loaded at the centre by load W.

In this case, it is given as; $\delta = Wl^3/4bd^3Y$, where δ is the sag or the measure of bending, Y is Young's modulus of elasticity. Study of beams is very useful in civil engineering and other such avenues. Using the above equation we can easily say that to reduce the amount of bending for a certain load, Young's modulus of elasticity of the material used must be large.

Also, the depth d must be considered since sag is inversely proportional to the cube of depth. But the problem faced on increasing the depth is that bending increases and this is known as buckling. Therefore, a compromise is made between the different cross-sectional shapes.

Solved Example For You

Q: Why is steel used in the construction of bridges?

Answer: Amongst bridge materials steel has the highest and most favourable strength qualities, and it is, therefore, suitable for the most daring bridges with the longest spans. Normal building steel has compressive and tensile strengths of 370 N/sq mm, about ten times the compressive strength of a medium concrete and a hundred times its tensile strength. A special merit of steel is its ductility due to which it deforms considerably before it breaks because it begins to yield above a certain stress level.

Stress and Strain

You must have noticed that there are certain objects that you can stretch easily. Let's say a rubber band. However, can you stretch an iron rod? Sound's impossible right? Why? In this chapter, we will look at these properties of solids in greater detail. We will see how quantities like stress can help us guess the strength of solids.

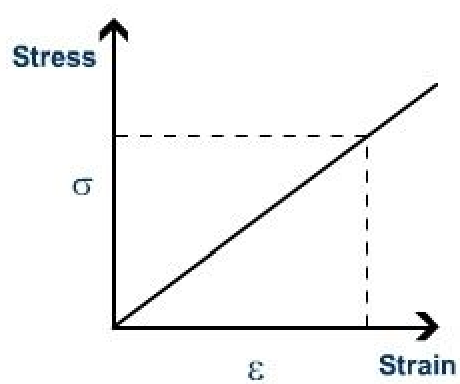
Properties of Solids

Intermolecular Force

In a solid, atoms and molecules are arranged in a way that neighbouring molecules exert a force on each other. These forces are intermolecular forces.

Elasticity

A body regains its original configuration (length, shape or volume) after you remove the deforming forces. This is elasticity.



Perfectly Elastic Body

A perfectly elastic body regains its original configuration immediately and completely after the removal of deforming force from it. Quartz and phosphor bronze are the examples of nearly perfectly elastic bodies.

Plasticity

A plastic body is unable to return to its original size and shape even on removal of the deforming force.

Stress

It is the ratio of the internal force F, produced when the substance is deformed, to the area A over which this force acts. In equilibrium, this force is equal in magnitude to the externally applied force. In other words,

The SI Unit of stress is newton per square meter (Nm^{-2}).In CGS units, stress is measured in dyne-cm^{-2}. Dimensional formula of stress is $ML^{-1}T^{-2}$

Types of Stress

- Normal stress: It is the restoring force per unit area perpendicular to the surface of the body. It is of two types: tensile and compressive stress.

- (Tangential stress: When the elastic restoring force or deforming force acts parallel to the surface area, the stress is called tangential stress.

Strain

It is the ratio of the change in size or shape to the original size or shape. It has no dimensions, it is just a number.

Types of Strain

- Longitudinal strain: If the deforming force produces a change in length alone, the strain produced in the body is called longitudinal strain or tensile strain. It is given as:

- Volumetric strain: If the deforming force produces a change in volume alone, the strain produced in the body is called volumetric strain. It is given as:

- Shear strain: The angle tilt caused in the body due to tangential stress expressed is called shear strain. It is given as:

The maximum stress to which the body can regain its original status on the removal of the deforming force is called the elastic limit.

Hooke's Law

Hooke's law states that, within elastic limits, the ratio of stress to the corresponding strain produced is a constant. This constant is called the modulus of elasticity. Thus,

Stress-Strain Curve

Stress-strain curves are useful to understand the tensile strength of a given material. The given figure shows a stress-strain curve of a given metal.

- The curve from O to A is linear. In this region, the material obeys the Hooke's Proportional limit law.

- In the region from A to C stress and strain are not proportional. Still, the body regains its original dimension, once we remove the load.
- Point B in the curve is the yield point or elastic limit and the corresponding stress is the yield strength of the material.
- The curve beyond B shows the region of plastic deformation.
- The point D on the curve shows the tensile strength of the material. Beyond this point, additional strain leads to fracture, in the given material.

Solved Example For You

Q: A and B are two steel wires and the radius of A is twice that of B, if we stretch them by the same load, then the stress on B is:

A. Four times that of A
B. Two times that of A
C. Three times that of A
D. Same

Sol: A) Since stress is inversely proportional to the area, and the area is proportional to the square of the radius, then we can write: [Strees on B]/[Stress on A] = [Radius of A]2/[Radius of B]2. Since the radius of A is = 2 times the radius of B, the ratio in the above equation will be equal to 4.

Elastic Moduli

In the stress-strain curve given below, the region within the elastic limit (region OA) is of importance to structural and manufacturing sectors since it describes the maximum stress a particular material can take before being permanently deformed. The modulus of elasticity is simply the ratio between stress and strain. Elastic Moduli can be of three types, Young's modulus, Shear modulus, and Bulk modulus. In this article, we will understand elastic moduli in detail.

Elastic Moduli – Young's Modulus

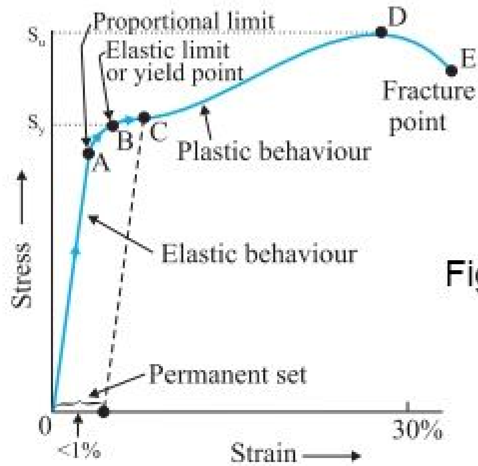


Fig. 1

Many experiments show that for a given material, the magnitude of strain produces is the same regardless of the stress being tensile or compressive. Young's modulus (Y) is the ratio of the tensile/compressive stress (σ) to the longitudinal strain (ε).

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$Y =$

σ

ε

… (1)

We already know that, the magnitude of stress =

F

A

and longitudinal strain =

ΔL

L

. Substituting these values, we get

$Y =$

F

A

ΔL

L

$\therefore Y =$

$(F \times L)$

$(A \times \Delta L)$

… (2)

Now, Strain is a dimensionless quantity. Hence, the unit of Young’s modulus is N/m2 or Pascal (Pa), the same as that of stress. Let’s look at Young’s moduli and yield strengths of some materials now:

Materials	Young’s Modulus Y (10^9 N/m^2)	Elastic Limit (10^7 N/m^2)	Tensile Strength (10^7 N/m^2)
Aluminum	70	18	20
Copper	120	20	40
Wrought Iron	190	17	33

Steel	200	30	50
Bone			
Tensile	16	–	12
Compressive	9	–	12

From the table, you can observe that Young's moduli for metals are large. This means that metals require a large force to produce a small change in length. Hence, the force required to increase the length of a thin wire of steel is much larger than that required for aluminum or copper. Therefore, steel is more elastic than the other metals in the table.

Determination of Young's Modulus of the Material of a Wire

The figure below shows an experiment to determine Young's modulus of a material of wire under tension.

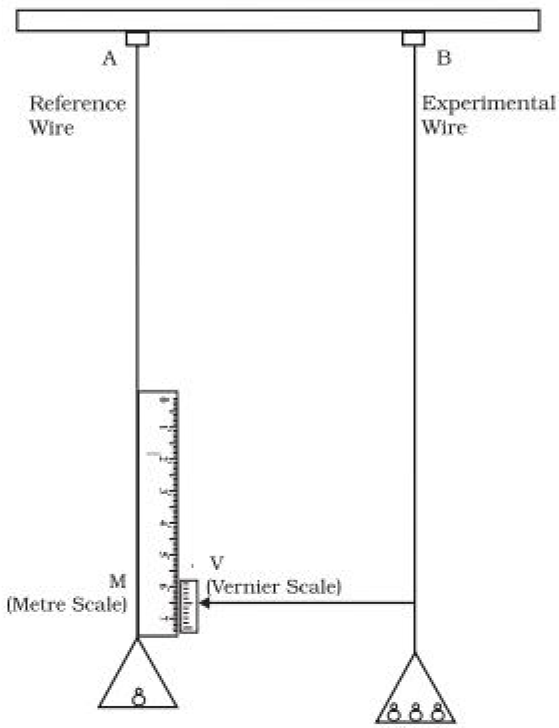

Fig. 2 *An arrangement for the determination of Young's modulus of the material of a wire.*

As can be seen in the diagram above, the setup consists of two long and straight wires having the same length and equal radius. These wires are suspended side-by-side from a fixed rigid support. The reference wire (wire A) has a millimeter main scale (M) and a pan to place weight.

The experimental wire (wire B) also has a pan in which we can place weights. Further, a vernier scale is attached to a pointer at the bottom of wire B and the scale M is fixed to reference wire A. Now, we place a small weight in both the pans to keep the wires straight and note the vernier scale reading.

Next, the wire B is slowly loaded with more weights, bringing it under tensile stress and the vernier reading is noted. The difference between the two readings gives the elongation produced in the wire. The reference wire A is used to compensate for any change in length due to a change in the temperature of the room.

Let r and L be the initial and final length of the wire B, respectively. Therefore, the area of the cross-section of the wire B is = πr2. Now, let M be the mass that produces an elongation of ΔL in wire B. Therefore, the applied force is = Mg, where 'g' is the acceleration due to gravity. Hence, using equations (1) and (2), Young's modulus of the material of wire B is:

Y =

σ

ε

=

$$\frac{Mg}{\pi r^2} \cdot \frac{L}{\Delta L}$$

$$\Rightarrow Y = \frac{(Mg \times L)}{(\pi r^2 \times \Delta L)} \quad \ldots (3)$$

Elastic Moduli – Shear Modulus

Shear Modulus (G) is the ratio of shearing stress to the corresponding shearing strain. Another name for shear stress is the Modulus of Rigidity.

$\therefore G = \frac{\text{shearing stress } (\sigma_s)}{\text{shearing strain}}$

$$\Rightarrow G = \frac{\frac{F}{A}}{\frac{\Delta x}{L}} = \frac{F \times L}{A \times \Delta x} \quad \ldots (4)$$

We also know that, Shearing strain = θ

$\therefore G =$

$$\frac{F/A}{\theta} = \frac{F}{A\times\theta} \quad \ldots (5)$$

Further, the shearing stress σ_s can also be expressed as

$$\sigma_s = G \times \theta \quad \ldots (6)$$

Also, the SI unit of shear modulus is N/m^2 or Pa. The shear moduli of a few common materials are given in the table below.

Material	Shear Modulus (G) 10^9 N/m^2
Aluminum	25

Brass	36
Copper	42
Glass	23
Iron	70
Lead	5.6
Nickel	77
Steel	84
Tungsten	150
Wood	10

From the table, you can observe that the shear modulus is less than Young's modulus for the same materials. Usually, G ≈

Y

3

.

Elastic Moduli – Bulk Modulus

We have already studied that when we submerge a body in a fluid, it undergoes a hydraulic stress which decreases the volume of the body, leading to a volume strain. Bulk modulus (B) is the ratio of hydraulic stress to the corresponding hydraulic strain.

B = -

p

(

ΔV

V

)

… (7)

The negative sign means that as the pressure increases, the volume decreases. Hence, for any system in equilibrium, B is always positive. The SI unit of the bulk modulus is N/m^2 or Pa. The bulk moduli of a few common materials are given in the table below.

Material	Bulk Modulus (B) 10^9 N/m^2
Aluminum	72
Brass	61
Copper	140
Glass	37
Iron	100
Nickel	260

Steel	160
Liquids	
Water	2.2
Ethanol	0.9
Carbon disulfide	1.56
Glycerine	4.76
Mercury	25
Gases	
Air (at STP)	1.0×10^{-4}

Compressibility (k) is the reciprocal of the bulk modulus. It is the fractional change in volume per unit increase in pressure.

$$\therefore k = \frac{1}{B} = -\frac{1}{\Delta p} \times \frac{\Delta V}{V} \quad \ldots (8)$$

From the table, you can observe that the bulk modulus for solids is much larger than that for liquids and gases. Hence, solids are the least compressible while gases are the most compressible. This is because, in solids, there is a tight coupling between the neighboring atoms.

Solved Examples for You on Elastic Moduli

Q1. A structural steel rod has a radius of 10 mm and a length of 1.0 m. A 100 kN force stretches it along its length. Calculate:

a. stress
b. elongation
c. the strain on the rod.

Young's modulus, of structural steel, is 2.0 × 1011 N/m2.

Answer: To solve the problem, let's assume that the rod is clamped at one end and a force F is applied at the other. This force is parallel to the length of the rod. Therefore, the stress on the rod is:

Stress =

F

A

=

F

π

r

2

=

100×

10

3

N

3.14×(

10

−2

m)

= 3.18 × 10^8 N/m^2

Next, let's calculate the elongation ΔL.

ΔL =

(

F

A

)L

Y

=

(3.18×

10

8

N

m

−2

)(1m)

2×

10

11

N

m

−2

$= 1.59 \times 10^{-3}$ m

$\therefore \Delta L = 1.59$ mm

Hence, Strain =

$$\frac{\Delta L}{L} = \frac{1.59 \times 10^{-3} \text{ m}}{1 \text{m}}$$

$= 1.59 \times 10^{-3}$

This is equivalent to 0.16%.

Hooke's Law and Stress-strain Curve

By now, we know that the stress and strain take different forms in different situations. In this article, we will understand the relationship between stress and strain by looking at the Hooke's law and the stress-strain curve.

The following figure shows some examples.

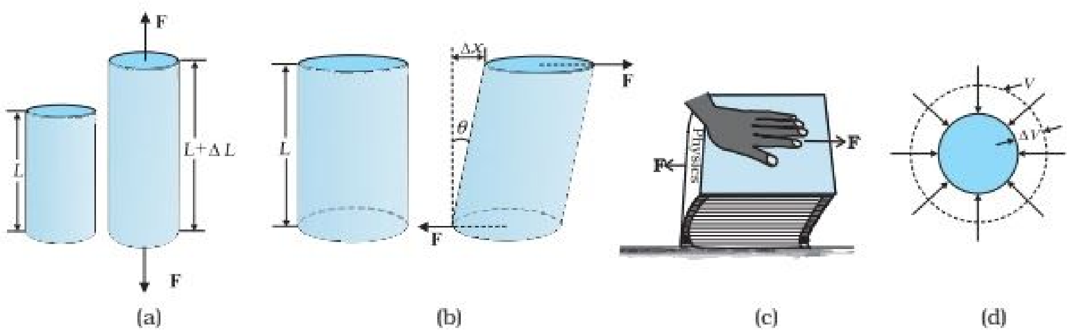

Fig. 1 *(a) Cylinder subjected to tensile stress stretches it by an amount ΔL. (b) A cylinder subjected to shearing (tangential) stress deforms by an angle θ .(c) A book subjected to a shearing stress (d) A solid sphere subjected to a uniform hydraulic stress shrinks in volume by an amount ΔV.*

Hooke's Law

Hooke's Law states that for small deformities, the stress and strain are proportional to each other. Thus,

Stress

$\propto$

Strain

Or, Stress = k × Strain … where k is the constant of proportionality and is the Modulus of Elasticity. It is important to note that Hooke's Law is valid for most materials.

Stress-Strain Curve

To determine the relation between the stress and strain for a given material, let's conduct an experiment. Take a test cylinder or wire and stretch it by an applied force. Record the fraction change in length (strain) and the applied force needed to cause the strain. Increase the applied force gradually, in steps, and record the readings.

Now, plot a graph between the stress (which is equal in magnitude to the applied force per unit area) and the strain produced. The graph for a typical metal looks as follows:

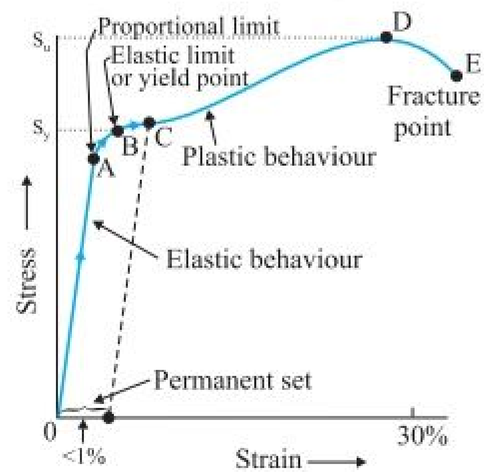

Fig. 2

The stress-strain curves can vary with the material in question. With the help of such curves, we can understand how the material deforms with increasing loads.

Analysis of the Curve

In Fig. 2, we can see that in the region between O and A, the curve is linear. Hence, Hooke's Law obeys in this region. In the region from A to B, the stress and strain are not proportional. However, if we remove the load, the body returns to its original dimension.

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The point B in the curve is the Yield Point or the elastic limit and the corresponding stress is the Yield Strength (S_y) of the material. Once the load is increased further, the stress starting exceeding the Yield Strength. This means that the strain increases rapidly even for a small change in the stress.

This is shown in the region from B to D in the curve. If the load is removed at, say a point C between B and D, the body does not regain its original dimension. Hence, even when the stress is zero, the strain is not zero and the deformation is called plastic deformation.

Further, the point D is the ultimate tensile strength (S_u) of the material. Hence, if any additional strain is produced beyond this point, a fracture can occur (point E). If,

- The ultimate strength and fracture points are close to each other (points D and E), then the material is brittle.

- The ultimate strength and fracture points are far apart (points D and E), then the material is ductile.

Exceptions

Remember, the stress-strain behavior varies from material to material. Rubber, for example, can be stretched up to several times its original length and it still returns to its original shape. The figure below shows the stress-strain curve for the elastic tissue or aorta, resent in the heart.

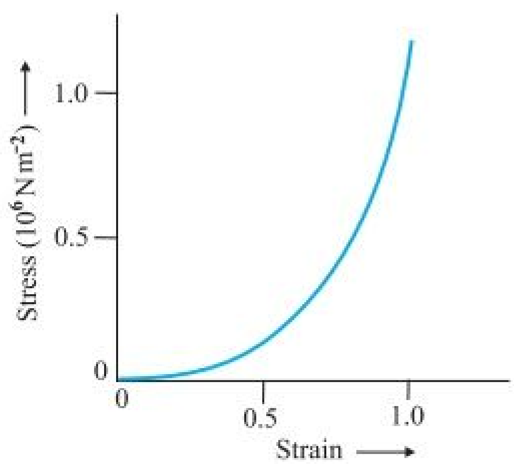

Fig. 3 *Stress-strain curve for the elastic tissue of Aorta, the large tube (vessel) carrying blood from the heart.*

From the curve, you can observe that while the elastic region is very large, the material does not obey Hooke's Law. Also, there is no well-defined plastic region. Materials like rubber, tissue or the aorta, etc. which can be stretched to cause large strains are called elastomers.

Solved Question For You

Q. Hooke's law essentially defines

A. Stress
B. Strain
C. Yield Point
D. Elastic Limit

Solution: Hooke's law is a principle which states that the force needed to extend or compress a spring by some distance is proportional to that distance. This proportionality constant defines the elastic limit.