Introduction to Rotational Dynamics

Have you ever wondered why tornadoes are so devastating? Is it the speed of the cyclone that engulfs the surroundings or there is something else to it! Well, a tornado is a mixture of force, power, and energy. These govern the rotational motion of a tornado, resulting in destructions. The chapter below is an introduction to rotational motion which will let you know how these three, effect the rotational motion of an object.

Rotational Dynamics

We come across many objects that follow rotational movements. No matter whether fixed or moving, these objects follow special dynamism which lets them perform their specific activity. Whether it is a ceiling fan or a potter's wheel, these rotating objects are a system of particles that consider the motion as a whole. In the introduction to rotational dynamics of a system, we shall emphasize on the center of mass of that particle and use the same in understanding motion as a whole.

Source: commons.wikimedia

Before going deeper into the subject, we should first understand the term "Extended body". When we refer to an object as an extended body we intend to signify it as a system of particles. Rigid bodies are those bodies with definite shape and size. In rigid bodies, the distance between the constituting pairs of particles does not change.

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- Motion of Centre of Mass
- Moment of Inertia
- Theorems of Parallel and Perpendicular Axis
- Rolling Motion

- Angular Velocity and Angular Acceleration
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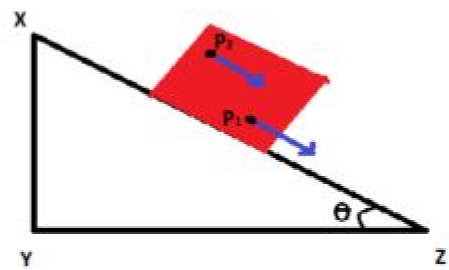
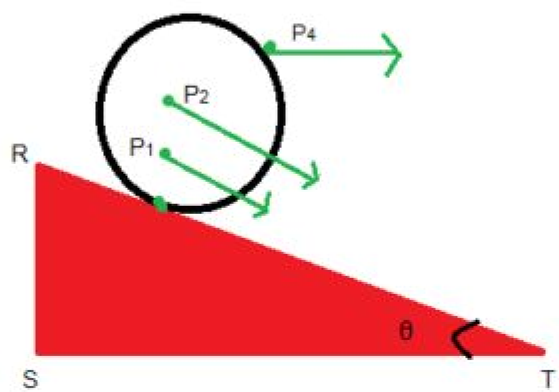
The motion of a Rigid Body

Let's consider a rigid body sliding down an inclined plane. The motion of this rigid body is in one direction, signifying therefore that all the particles are moving in a single direction. These particles are moving with the same velocity at any time interval. When all the particles in a system move with the same velocity at any instant of time, then such motion is called the translational motion.



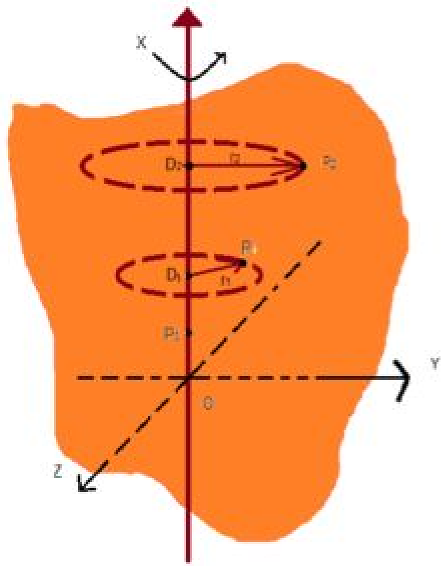
After translational motion, in our introduction to rational dynamics, we consider the rotational motion. For this, the figure below needs to be studied:



A cylinder when rolled down an inclined plane follows translational and rotational motion. Some of its particles move in the same direction while others follow a different path. To ensure their direction of motion we need to fix the movement of this cylinder body across a straight line. This straight line along which the motion of the cylinder is fixed and is called the axis of rotation. The circular motion of the cylinder is termed the rotational motion.

Rotation and Its Characteristics

In rotational motion, we know that the particles of the object, while moving follow a circular path. Every particle in the rigid body moves in a circular path along a plane that is perpendicular to the axis and has its center on the same axis. There can be two instances of rotational motion one about the fixed axis and second about an unfixed axis. The examples of rotation around a fixed axis are the fan while for unfixed axis the spinning top makes a perfect example. Here we will study rotation across a fixed axis.



The figure above shows a rigid body's rotation along a fixed axis. Here the axis on which the rotational motion occurs is the X-axis. If P1 is a particle of the rigid body's system moving with its center D1 on the fixed axis then the circular plane in which the motion takes place lies in a plane that is perpendicular to the X-axis. The same situation is with another particle P2. Here both these particles move in a circular motion with their respective radius r1 and r2 at centers D1 and D2.

The point to be noted here is that though these two particles P1 and P2 move in a plane perpendicular to the X-axis, their point of rotation is different from each other. Now another particle P3 seems stationary as its r = 0. Therefore while studying an object's motion, we sum up the movement of various particles to reach a conclusion. In rotational motion, different particles show different points of motion.

Summary

These cases where the point of the axis is not fixed for example a spinning top, we know that at a vertical point the spin is fixed. This vertical point where the top is fixed to the ground is taken as the axis of rotation. This implies that in our introduction to rotational dynamics we take every rigid body showing rotational motion as moving on a fixed axis.

Henceforth we come to a conclusion that motions basically are of two types, translational and rotational. The movement of a rigid body not fixed or pivoted shows a translation motion while a body with a fixed axis shows a combination of translational and rotational motion.

Comparison Between Translational and Rotational Motion

- Objects showing translational motion move with constant velocity. Objects showing rotational motion move with an angular velocity. Both these velocities are constant unless changed externally.

- In the translational motion, the acceleration is inversely proportional to mass and directly proportional to the force. In rotational motion, force is substituted by torque. Acceleration, in this case, is termed as angular acceleration.

Solved Examples For You

Consider a string with several rocks tied along its length at equally spaced intervals. You whirl the string overhead so that the rocks follow circular paths. Compared to a rock in the middle of the string, a rock at the outer end moves

A. Half as fast
B. Twice as fast
C. At the same linear speed
D. Can't say

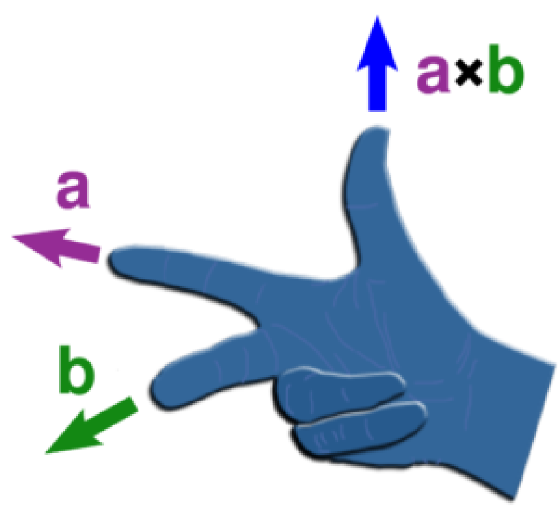
Solution: B) Since the angular frequency of each rock is same, from $v = \omega r$, we can see that the rock in the middle has a rotational radius only half that of the rock in the far end. In other words, we can say that the velocity of the rock at the outer end would be twice that of the velocity of the rock that is present inside.

Vector Product of Two Vectors

We know that a vector has magnitude as well as a direction. But do we know how any two vectors multiply? Let us now study about the cross product of these vectors in detail.

Vector Product of Two Vectors

Vector product also means that it is the cross product of two vectors.



If you have two vectors a and b then the vector product of a and b is c.

$$c = a \times b$$

So this $a \times b$ actually means that the magnitude of $c = ab \sin\theta$ where θ is the angle between a and b and the direction of c is perpendicular to a well as b. Now, what should be the direction of this cross product? So to find out the direction, we use the rule which we call it as the "right-hand thumb rule".

Suppose we want to find out the direction of $a \times b$ here we curl our fingers from the direction of a to b. So if we curl our fingers in a direction as shown in the above figure, your thumb points in the direction of c that is in an upward direction. This thumb denotes the direction of the cross product.

While applying rules to direction, the rotation should be taken to smaller angles that is $<180°$ between a and b. So the fingers should always be curled in acute angle between a and b.

Properties of Vector Cross Product

1] *Vector product is not commutative.* That means $a \times b \neq b \times a$

We saw that a × b = c here the thumb is pointing in an upward direction. Whereas in b × a the thumb will point in the downward direction. So, b × a = – c. So it is not commutative.

2] *There is no change in the reflection.*

What happens to a × b in the reflection? Suppose vector a goes and strikes the mirror, so the direction of a will become – a. So under reflection, a will become – a and b will become – b. Now a × b will become -a × -b = a × b

3] *It is distributive with respect to vector addition.*

This means that if a × (b + c) = a × b + a × c. This is true in case of addition.

Vector Product of Unit Vectors

The three unit vectors are

i

^

,

j

^

and

k

^

. So,

- i
- ^
- ×
- i
- ^
- = 0

- i
- ^
- ×
- j
- ^
- = 1
- k
- ^

-
- i
- ^
- ×
- k
- ^
- =1 –
- j
- ^
-
- j
- ^
- ×
- i
- ^
- = –
- k
- ^
-
- j
- ^
- ×
- j
- ^
- = 0
- j
- ^
- ×

- k
- ^
- = 1
- i
- ^
-
- k
- ^
- ×
- i
- ^
- =
- j
- ^
-
- k
- ^
- ×
- j
- ^
- = -
- i
- ^
-
- k
- ^
- ×
- k

- ^
- = 0

This is how we determine the vector product of unit vectrors.

Mathematical Form of Vector Product

a = a_x

i

^

+ a_y

j

^

+a_z

k

^

$b = b_x$

i

$\wedge$

$+ b_y$

j

$\wedge$

$+ b_z$

k

$\wedge$

$a \times b = (\ a_x$

i

$\wedge$

$+ a_y$

j

^

$+a_z$

k

^

$) \times (b_x$

i

^

$+ b_y$

j

^

$+b_z$

k

^

)

$$= a_x \hat{i} \times (b_x \hat{i} + b_y \hat{j} + b_z \hat{k}) + a_y \hat{j}$$

^

× (b_x

i

^

+ b_y

j

^

+b_z

k

^

) + a_z

k

^

× (b_x

i

^

+ b_y

j

^

+b_z

k

^

)

= a_x b_y

k

^

– a_x b_z

j

^

$+ a_y b_z$

i

^

$+ a_z b_x$

j

^

$- a_z b_y$

i

^

$a \times b = (a_y b_z - a_z b_y)$

i

^

$+ (a_z b_x - a_x b_z)$

j

^

$+ (a_x b_y - a_y b_x)$

k

^

So the determinant form of the vectors will be, $a \times b =$

i	j	k
^	^	^
a_x	a_y	a_z
b_x	b_y	b_z

Solved Question

Q1. The magnitude of the vector product of two vectors

P

→

and

Q

→

may be:

A. Equal to PQ

B. Less than PQ

C. Equal to zero

D. All of the above.

Answer: The correct option is “D”. |

P

→

×

Q

→

=

P

→

Q

→

sinθ, where θ is the angle between P and Q.

Centre of Mass

Complex objects have particles that show mechanism differently. When we work on a system of particles we need to know its centre of mass to calculate the mechanics of oddly shaped objects. Rigid bodies constitute a system of particles which govern its motion and equilibrium. With the Centre of Mass, we can effortlessly understand the mechanism of complicated objects. Let’s see how?

Suggested Videos

Centre of mass and Linear momentum

Cross Product of two vectors in Rectangular Coordinate System

Problem on Moment of Inertia

Centre of Mass

[source: psychologywizard]

The centre of mass (CoM) is the point relative to the system of particles in an object. This is that point of the system of particles that embarks the average position of the system in relation to the mass of the object. At the centre of mass, the weighted mass gives a sum equal to zero. It is the point where any uniform force applied on the object acts.

In other words, a particle's centre of mass is the point where Newton's law of motions applies perfectly. When force is applied to the centre of mass, the object as a system of particles moves in the direction of force without rotating. No matter what the shape of the object, the centre of mass helps understand the mechanism of force and motion of that object.

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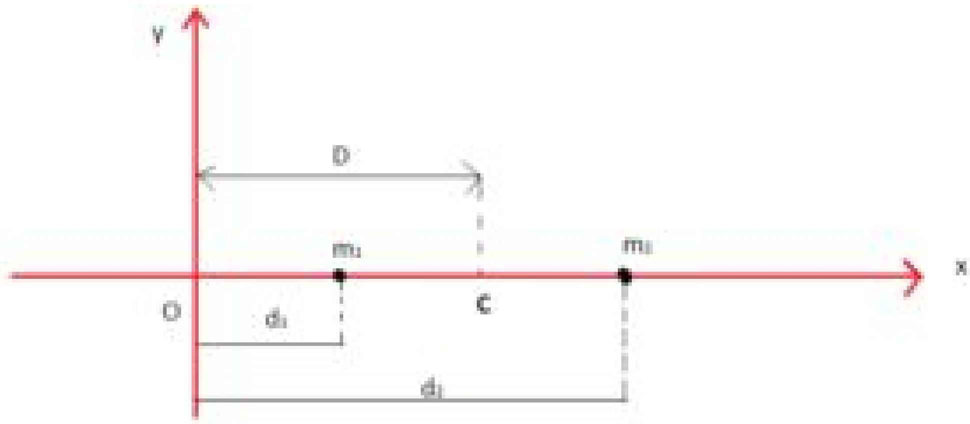
Centre of Mass for Two Particles

For a system of two particles with equal masses, CoM is the point that lies exactly in the middle of both.



In the figure above, we take two particles with masses m_1 and m_2 respectively lying on the x-axis. The distance of both the particles from the centre O is d_1 and d_2 respectively. The CoM of this two system of particles is at point C lying at a distance D from point O. In this system of two particles D can be written as:

$$D = m_1d_1 + m_2d_2 / m_1 + m_2$$

From the equation D, can be taken as the mass-weighted mean of d1 and d2. Now, let us presume that the particles in the system have equal masses. Hence, $m_1=m_2 = m$, in this case,

$$D = (md_1 + md_2)/ 2m = m (d_1 + d_2) / 2m$$

$$D = (d_1 + d_2) / 2$$

From the equation above we get the centre of mass of two particles with equal masses. From the above equation, it is clear that the CoM of two particles lies in the midway of both.

Centre of Mass for n Particles

For a system of n particles, the centre of mass, according to its definition is:

$$D = m_1d_1 + m_2d_2 + m_3d_3 + \ldots\ldots m_nd_n / m_1 + m_2 + m_3 + \ldots\ldots m_{nm} = \sum m_id_i / m_i$$

m_i here is the sum of the masses of the particles.

Centre of Mass for Three Particles at Different Positions

It is not necessary that the system of particles spoken about for the centre of mass lie on the same axis of a straight line. There may be cases when particles lie on different lines away from each other. What shall be the CoM in such case? Since these particles are not in a straight line hence there must be more than two particles.

Let’s take the case of three particles lying on different axes x and y. Here, we represent the positions of the particles as (d_1,e_1), (d_2,e_2) and (d_3,e_3) along x and y-axes. The CoM C for the system of three particles is located at D and E and are given as:

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Now, if the particles have equal masses $m_1 = m_2 = m_3 = m$

$$\text{then, } D = m\,(d_1 + d_2 + d_3) / 3m = d_1 + d_2 + d_3 / 3$$

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Centre of Mass for a System of Particles Outside a Plane

As said earlier, Centre of Mass helps in calculating the mechanism of forces for complex objects, which encompass particles lying at different positions. Now, if the object has a complex shape and the system of particles rather than lying in a plane are distributed variably in such situation calculating the centre of mass depends on the position of the particles.

Here, since the particles lie in different planes we take the CoM of such particles to lie on D, E and F. So,

$$D = \sum m_i d_i / M$$

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$\sum m_i$ here is the sum of the total mass of the system of particles, while m_i is the mass of i_{th} particle and the position of respective particles is given by d_i,e_i and f_i. Using this information of position vectors we combine the above equations, to get,

$$R = \sum m_i r_i / M$$

R is the position vector of the CoM and r_i is the position vector of the i_{th} particle.

Centre of Mass of Homogeneous Bodies

Homogeneous bodies are those objects which have a uniformly distributed mass around the body as a whole. A few examples for homogeneous bodies are spheres, rings etc. These rigid bodies have a regular shape and for calculating the CoM for these we keep in mind the symmetry between the particles. According to our symmetric considerations, we can assume that the CoM for these regular bodies lies at their geometric centres.

Solved Examples for You

Question: Two spheres of mass M and 7M are connected by a rod whose mass is negligible, and the distance between the centres of each sphere is d. How far from the centre of the 7M sphere is the Centre of Mass for this object?

A. d / 8
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Solution: Option (A) d/8. Point C represents the centre of mass of the system.

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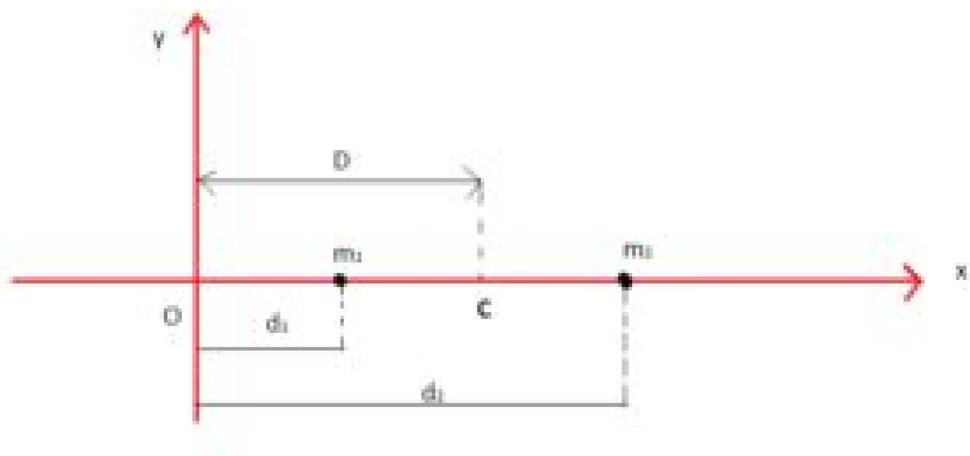

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$x = (7M) \times 0 + Md / 7M + M = d / 8$

Motion of Centre of Mass

Suppose you take 2 similar tennis balls and put them a bit apart, you see that the centre of mass of the two balls would be their centre. If one of these balls was heavier, the centre of mass will shift towards the heavier ball. But if you take a cricket bat, the centre of mass would be below the centre of the bat, in the lower half. Let us study more about the motion of centre of mass.

Motion of Center of Mass

Let us talk about centre of mass of a system of particles.

Whenever we talk about motion of an object, we usually talk about velocity with which the object is moving or the acceleration with which the object is moving. As we know the centre of mass is denoted by x and y.

$$X = m_i x_i / M$$

$$Y = m_i y_i / M$$

X = Σ

m

i

x

i

$Y = \Sigma$

m

i

y

i

The position vector of the centre of mass can be written as

Σ

m

i

r

i

M

$\Rightarrow m_1 r_1 + m_2 r_2 + \ldots\ldots\ldots. + m_n r_n$

Differntiating on both the sides,

$$M \frac{dr}{dt} = M_1 \frac{dr_1}{dt} + M_2 \frac{dr_2}{dt} + M_n$$

d

r

n

dt

Change of displacement of time is the velocity.

$$mv = m_1 v_1 + m_2 v_2 + \ldots\ldots\ldots.. m_n v_n$$

where v_1 is the velocity of the particle and v =

dr

dt

is the velocity of centre of mass.

V = Σ

m

i

v

i

M

This is an expression for velocity of centre of mass.

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Acceleration of System of Particles

Now we know that, $mv = m_1 v_1 + m_2 v_2 + \ldots\ldots\ldots\ldots + m_n v_n$,

differentiating on both the sides we get,

M

dv

dt

= M_1

d

v

1

dt

= M_2

d

v

2

dt

$+ \ M_n$

d

v

n

dt

Now the rate of change of velocity is,

$$MA = m_1a_1 + m_2a_2 + \ldots\ldots + m_na_n$$

A =

dV

dt

is the acceleration of centre of mass of system of particles. The force of particle is given by F So, $MA = F_1 + F_2 + \ldots + F_n$

$$MA = F_{ext}$$

which is the sum of all external forces acting on the particles of the system. This is how we determine the velocity and acceleration of the centre of mass of the system of particles. Hence we conclude that the centre of the mass of the system of particles moves as if all the mass system was concentrated at the centre of mass and all external forces were applied to that point.

Questions For You

Q1. Two objects P and Q initially at rest move towards each other under a mutual force of attraction. At the instant when the velocity of P is V and that of Q is 2v the velocity of the centre of mass of the system is

A. v
B. 2v
C. 3v
D. 1.5v
E. Zero

Solution: E. Since they move due to the mutual interaction between two objects so, the centre of mass remains the same and its velocity is zero.

Q2. A ball kept in a closed container moves in it making collision with the walls. The container is on a smooth surface. The velocity of the centre of mass of

A. the ball remains fixed.
B. ball relative to container remains the same.
C. container remains fixed.
D. ball container and ball remains fixed.

Solution: D. Since the container is closed, the collisions made by the ball with the walls of a container will not affect the mass of container and in turn, there is no change in velocity of the centre of mass.

Moment of Inertia: Formula, Definition, and Examples

Moment of inertia aka angular mass or rotational inertia can be defined w.r.t. rotation axis, as a quantity that decides the amount of torque required for a desired angular acceleration or a property of a body due to which it resists angular acceleration. The formula for moment of inertia is the "sum of the product of mass" of each particle with the "square of its distance from the axis of the rotation". The formula of Moment of Inertia is expressed as $I = \Sigma m_i r_i^2$.

Moment of Inertia Example

Imagine you are on a bus right now. You find a seat and sit down. The bus starts moving forward. After a few minutes, you arrive at a bus stop and the bus stops. What did you experience at this point? Yes. When the bus stopped, your upper body moved forward whereas your lower body did not move.

Why is that? It is because of Inertia. Your lower body is in contact with the bus but your upper body is not in contact with the bus directly. Therefore, when the bus stopped, your lower body stopped with the bus but your upper body kept moving forward, that is, it resisted change in its state.

Similarly, when you board a moving train, you experience a force that pushes you backward. That is because before boarding the train you were at rest. As soon as you board the moving train, your lower body comes in contact with the train but your upper body is still at rest.

Therefore, it gets pushed backward, that is, it resists change in its state.

Understand the Theorem of Parallel and Perpendicular Axis here in detail.

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What is Inertia?

What is Inertia? It is the property of a body by virtue of which it resists change in its state of rest or motion. But what causes inertia in a body? Let's find out.

Inertia in a body is due to it mass. More the mass of a body more is the inertia. For instance, it is easier to throw a small stone farther than a heavier one. Because the heavier one has more mass, it resists change more, that is, it has more inertia.

Moment of Inertia Definition

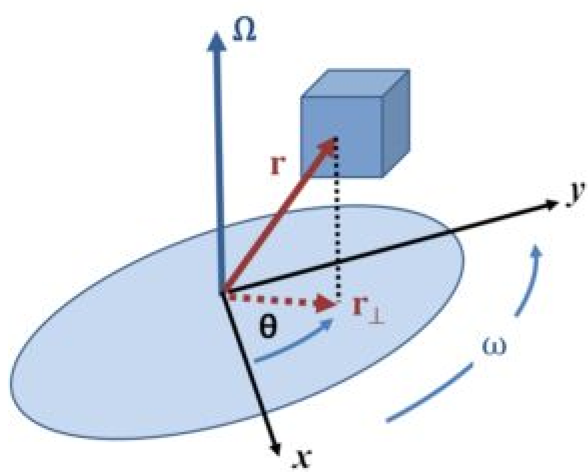
So we have studied that inertia is basically mass. In rotational motion, a body rotates about a fixed axis. Each particle in the body moves in a circle with linear velocity, that is, each particle moves with an angular acceleration. Moment of inertia is the property of the body due to which it resists angular acceleration, which is the sum of the products of the mass of each particle in the body with the square of its distance from the axis of rotation.

Formula for Moment of Inertia can be expressed as:

$$\therefore \text{Moment of inertia } I = \Sigma m_i r_i^2$$

Kinetic Energy in Rotational Motion

What is the analogue of mass in rotational motion? To answer this question, we have to derive the equation of kinetic energy in rotational motion.



Consider a particle of mass *m* at a distance from the axis with linear velocity = $v_i = r_i\omega$. Therefore, the kinetic energy of this particle is,

$$k_i = 1/2\ m_i v_i^2 = 1/2\ m_i (r_i)^2 \omega^2$$

where m_i is the mass of the particle. The total kinetic energy K of the body is thus the sum of the kinetic energies of individual particles. ∴ $K = \Sigma k_i = 1/2 (\Sigma m_i(r_i)^2\omega^2)$ where n is the number of particles in the body. We know that angular acceleration ω is the same for all particles. Therefore, taking ω out of the sum, we get,

$$K = 1/2\omega^2 (\Sigma m_i(r_i)^2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots(I)$$

Let $\Sigma m_i(r_i)^2$ be I which is a new parameter characterising the rigid body known as the Moment of Inertia. Therefore, with this definition,

$$K = 1/2I\omega^2 \ldots\ldots\ldots\ldots\ldots(II)$$

The parameter I is independent of the magnitude of the angular velocity. It is the characteristic of the rigid body and the axis about which it rotates. We already know that linear velocity in linear motion is analogous to angular acceleration in rotational motion.

On comparing equation II with the formula of the kinetic energy of the whole rotating body in linear motion, it is evident that mass in linear motion is analogous to the moment of inertia in rotational motion. Hence, the question is answered. In simple words, moment of inertia is the measure of the way in which different parts of the body are distributed at different distances from the axis.

Radius of Gyration

As a measure of the way in which the mass of a rotating rigid body is distributed with respect to the axis of rotation, we define a new parameter known as the radius of gyration. It is related to the moment of inertia and the total mass of the body. Notice that we can write $I = Mk^2$ where k has the dimension of length.

Therefore, the radius of gyration is the distance from the axis of a mass point whose mass is equal to the mass of the whole body and whose moment of inertia is equal to the moment of inertia of the body about the axis. Therefore, the moment of inertia depends not only on the mass, shape, and size of the body but also the distribution of mass in the body about the axis of rotation.

Learn more about Torque and Angular Momentum here.

Here's a Solved Question for You

Q: The moment of inertia depends on:

a) mass of the body b) shape and size of the body

c) distribution of mass about the axis of rotation d) all of the above

Solution: d) all of the above. All the factors together determine the moment of inertia of a body.

Theorems of Parallel and Perpendicular Axis

Suppose you have an object like a ball or a rotating disc that rotates about its centre. Now, if you move the point of rotation to the point other than its centre. How do you find the moment of inertia? The answer to this question is by the parallel axis theorem.

Before we study the theorems of parallel and perpendicular axis let us first see what moment of inertia is. Moment of inertia is the property of the body due to which it resists angular acceleration, which is the sum of the products of the mass of each particle in the body with the square of its distance from the axis of rotation.

∴ Moment of inertia $I = \Sigma\, m_i r_i^2$

So in order to calculate the moment of inertia we use two important theorems which are the perpendicular and parallel axis theorem.

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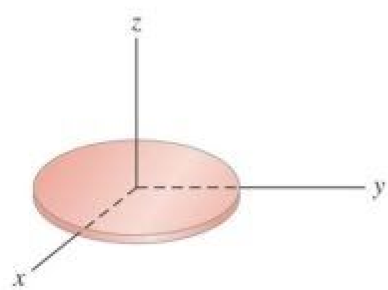
Perpendicular Axis Theorem

This theorem is applicable only to the planar bodies. Bodies which are flat with very less or negligible thickness. This theorem states that the moment of inertia of a planar body about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with the perpendicular axis and lying in the plane of the body.

$$I_z = I_x + I_y.$$



(Source: Toproadrunner5)

In the above figure, we can see the perpendicular body. So Z axis is the axis which is perpendicular to the plane of the body and the other two axes lie in the plane of the body. So this theorem states that

$$I_Z = I_x + I_y$$

That means the moment of inertia about an axis which is perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes.

Let us see an example of this theorem:

Suppose we want to calculate the moment of inertia of a uniform ring about its diameter. Let its centre be $MR^2/2$, where M is the mass and R is the radius. So, by the theorem of perpendicular axes, $I_Z = I_x + I_y$. Since the ring is uniform, all the diameters are equal.

$\therefore\ I_x = I_y$

$\therefore\ I_Z = 2\,I_x$

$I_z = MR^2/4$

So finally the moment of inertia of a disc about any of its diameter is $MR^2/4$

Learn more about Moment of Inertia in detail here.

Parallel Axis Theorem

Parallel axis theorem is applicable to bodies of any shape. The theorem of parallel axis states that the moment of inertia of a body about an axis parallel to an axis passing through the centre of mass is equal to the sum of the moment of inertia of body about an axis passing through centre of mass and product of mass and square of the distance between the two axes.

$$I_{Z'} = I_z + M\alpha^2$$

where, α is the distance between two axes.

Solved Examples For You

Q1. The moment of inertia of a thin uniform rod of mass M and length L bout an axis perpendicular to the rod, through its centre is I. The moment of inertia of the rod about an axis perpendicular to the rod through its endpoint is:

A. I/4

B. I/2

C. 2I

D. 4I

Answer: D. $I_{centre} = ML^2/12$ and $I_{endpoint} = ML^2/3 = 4I$

Q2. The radius of gyration of a body is 18 cm when it is rotating about an axis passing through a centre of mass of a body. If the radius of gyration of the same body is 30 cm about a parallel axis to the first axis then, the perpendicular distance between two parallel axes is:

A. 12 cm

B. 16 cm

C. 24 cm

D. 36 cm

Answer: C. $I_{CM} = MK^2$, where K is the radius of gyration of the body. K = 18 cm. Moment of inertia around the axis parallel to the axis passing through the centre of mass is:

$I = I_{CM} = Mx^2$

X is a perpendicular distance of the axis from CM axis and the radius of gyration for this new moment of inertia is 30 cm.

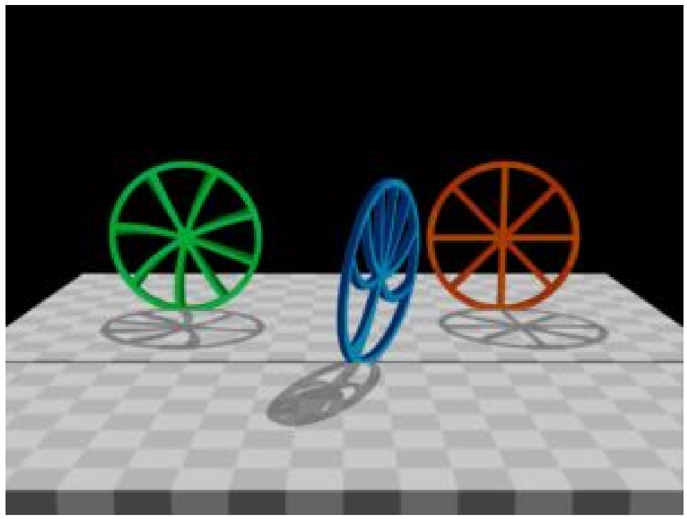
So, x = 24 cm

Rolling Motion

Every one of you must have seen a ball rolling down a hill or rolling of bike wheels when the bike is moving. You must have also seen the motion of a bowling ball or a motion of snooker ball on the table. These are nothing but the examples of a rolling motion. Now let us study this in detail.

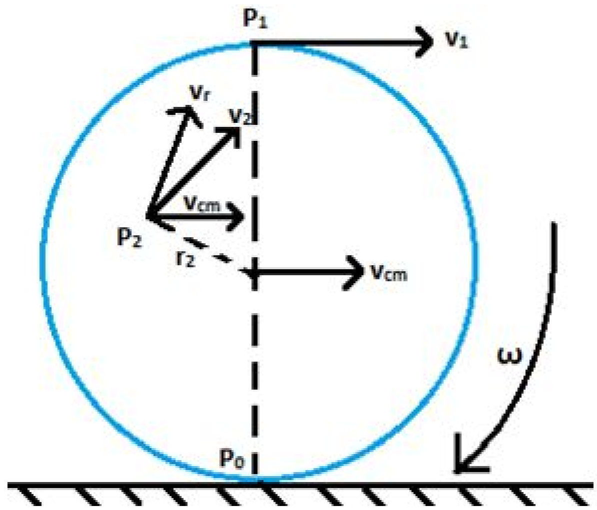
Rolling Motion

Let us understand the concept of "Rolling Motion". Imagine a box sliding down from an inclined plane. This is an example of a translation motion. They have the same velocity at an instant of time. The motion of a ceiling fan or a merry go round are the examples of rotational motion. Here in rotational motion, every particle of the body moves in a circle. Rolling motion is the combination of rotation and translation.



For example, an object, say a ball is in rolling, that is it is rolling on the surface of the ground. So the ball is in the rotation motion. At the same time, the ball is moving from one point to another point so there is a translation motion. At any instant of time, there is one point which is always in contact with the surface. So that point is at rest.

Rolling Motion of Disc



(Source: My Rank)

Let us assume that the disc rolls without slipping. If that this disc is the uniform disc the centre of mass lies at the centre of the disc that is the point '0'. The velocity of the centre of mass will always be parallel to the surface as we can see this in the figure.

At any instant of time, there will be two velocities. One is the velocity of the centre of mass and another one is the component of linear velocity. The velocity of the centre of mass is v_{cm}. This corresponds to the translation motion of the object. v_r is the linear velocity which corresponds to the rotation.

$v_r = r\omega$.

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Let us first consider the point P_0

So at P_0 we have two velocities that are v_{cm} and v_r. Here the direction of the centre of mass is in the direction of P_0 and v_{cm} and v_r are opposite in direction. If we want the object to roll without slipping, the point which is in contact with the ground should be at rest. So $v_r = v_{cm}$

$$\Rightarrow r\omega = v_{cm}$$

The above is the condition for rolling without slipping.

At point P_1

Also here there are two velocities that are v_{cm} and v_1. So the net velocity of P is,

$$v_1 = v_{cm} + v_r = v_{cm} + r\omega = r\omega + r\omega = 2r\omega$$

The same is true for all the points on the disc.

Kinetic Energy for Rolling Motion

Since rolling is the combination of rotational motion and translational motion,

$$K.E = K_T + K_R$$

=

$$\frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I\omega^2$$

I = moment of inertia. We can write moment of inertia as $I = mk^2$, k = radius of gyration.

=

1

2

$m\ v_{cm}^2 +$

1

2

$mk^2\ \omega^2$

For rolling without slipping the mathematical condition is $r\omega = v_{cm}$

K =

1

2

$m^2\ \omega^2 +$

1

2

$mk^2 \omega^2$

$=$

1

2

$m v_{cm}^2 +$

1

2

mk² \(\frac{v_cm²}{r²} \)

$\Rightarrow K =$

1

2

$m v_{cm}^2 [1 +$

k^2

r^2

]

This is the kinetic energy of a rolling motion.

Questions For You

Q. S_1 and S_2 are two spheres of equal masses. S_1 runs down a smooth inclined plane of length 5 cm and of height 4 cm and S_2 falls vertically down by 4 cm. the work was done by all forces acting on S_1

A. Is greater than that on S_2
B. and on S_2 are same and non zero
C. is less than that of S_2
D. as well as on S_2 is zero.

Solution: B. Using energy balance we see that potential energy which only depends on the height changes to kinetic energy at the bottom. Thus the work done in both the cases is same. If friction were present work done would have been different.

Angular Velocity and Angular Acceleration

Angular velocity is the rate of velocity at which an object or a particle is rotating around a center or a specific point in a given time period. It is also known as rotational velocity. Angular velocity is measured in angle per unit time or radians per second (rad/s). The rate of change of angular velocity is angular acceleration. Let us learn in more detail about the relation between angular velocity and linear velocity, angular displacement and angular acceleration.

Angular Velocity

Angular velocity plays an eminent role in the rotational motion of an object. We already know that in an object showing rotational motion all the particles move in a circle. The linear velocity of every participating particle is directly related to the angular velocity of the whole object.

These two end up as vector products relative to each other. Basically, the angular velocity is a vector quantity and is the rotational speed of an object. The angular displacement of in a given period of time gives the angular velocity of that object.

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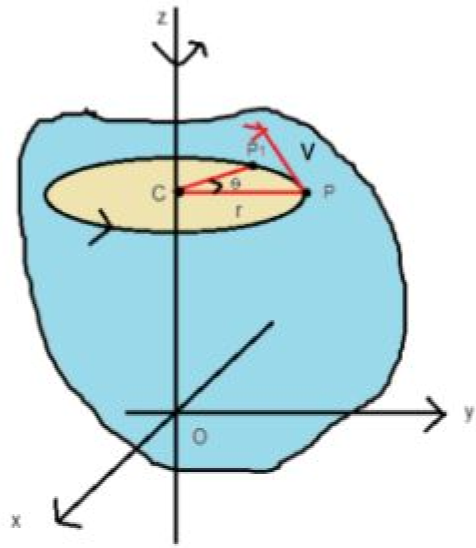
Relation Between Angular Velocity and Linear Velocity

For understanding the relation between the two, we need to consider the following figure:



The figure above shows a particle with its center of the axis at C moving at a distance perpendicular to the axis with radius r. v is the linear velocity of the particle at point P. The point P lies on the tangent of the circular motion of the particle. Now, after some time(Δt) the particle from P displaces to point P1. $\Delta\theta$ or $\angle PCP1$ is the angular displacement of the particle after the time interval Δt. The average angular velocity of the particle from point P to P1 = Angular displacement / Time Interval = $\Delta\theta/\Delta t$

At smallest time interval of displacement, for example, when Δt=0 the rotational velocity can be called an instantaneous angular (ω) velocity, denoted as dt/dθ for the particle at position P. Hence, we have ω = dt/dθ

Linear velocity (v) here is related to the rotational velocity (ω) with the simple relation, $v = \omega r$, r here is the radius of the circle in which the particle is moving.

Angular Velocity of a Rigid Body

This relation of linear velocity and angular velocity apply on the whole system of particles in a rigid body. Therefore for any number of particles; linear velocity $v_i = \omega r_i$

'i' applies for any number of particles from 1 to n. For particles away from the axis linear velocity is ωr while as we analyze the velocity of particles near the axis, we notice that the value of linear velocity decreases. At the axis since r=0 linear velocity also becomes a zero. This shows that the particles at the axis are stationary.

A point worth noting in case of rotational velocity is that the direction of vector ω does not change with time in case of rotation about a fixed axis. Its magnitude may increase or decrease. But in case of a general rotational motion, both the direction and the magnitude of angular velocity (ω) might change with every passing second.

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Rotational Velocity of Revolutions

When a rigid body rotates around an axis, after the lapse of some time it completes a revolution. The time taken by that rigid body to complete a revolution is called the frequency of that body. Rotational velocity and frequency hence have a relation between each other. Here one revolution is equal to 2π, hence $\omega = 2\pi / T$

The time taken to complete one revolution is T and $\omega = 2\pi f$. 'f' is the frequency of one revolution and is measured in Hertz.

Angular Acceleration

When an object follows a rotational path, it is said to move in an angular motion or the commonly known rotational motion. In the course of such motion, the velocity of the object is always changing. Velocity being a vector involves a movement of an object with speed that has direction. Now, since in a rotational motion, the particles tend to follow a circular path their direction at every point changes constantly. This change results in a change in velocity. This change in velocity with time gives us the acceleration of that object.

Angular acceleration is a non-constant velocity and is similar to linear acceleration of translational motion. Understanding linear displacement, velocity, and acceleration are easy and this is why when we intend to study rotational motion, we compare its vectors with translational motion. Like linear acceleration, angular acceleration (α) is the rate of change of angular velocity with time. Therefore, $\alpha = d\omega/dt$

Now since for rotation about a fixed axis the direction of angular velocity is fixed therefore the direction of angular momentum α is also fixed. For such cases, the vector equation transforms into a scalar equation.

Solved Question For You

1. The angular velocity of a scooter tire of diameter 6 inches rotates 6 times a second is:

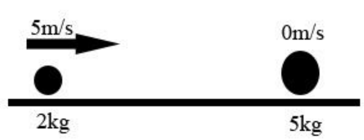
a. 16π b. 2π c. 12π d. none

Solution: c) 12π. We have: $\omega = 2\pi f$. The frequency of the tire is 6 revolution per second;

Therefore we can write, $\omega = 2\pi \times 6 = 12\pi$

Linear Momentum of a System of Particles

Linear momentum is a product of the mass (m) of an object and the velocity (v) of the object. If an object has higher momentum, then it harder to stop it. The formula for linear momentum is p = mv. The total amount of momentum never changes, and this property is called conservation of momentum. Let us study more about Linear momentum and conservation of momentum.



Linear Momentum of System of Particles

We know that the linear momentum of the particle is

$$p = mv$$

Newton's second law for a single particle is given by, F =

dP

dt

where F is the force of the particle. For ‘ n ‘ no. of particles total linear momentum is,

$$P = p_1 + p_2 + \ldots + p_n$$

each of momentum is written as $m_1 v_1 + m_2 v_2 + \ldots\ldots + m_n v_n$. We know that velocity of the centre of mass is V = Σ

m

i

v

i

M

,

$$mv = \Sigma\ m_i v_i$$

So comparing these equations we get,

$$P = M\ V$$

Therefore we can say that the total linear momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its center of mass. Differentiating the above equation we get,

dP

dt

= M

dV

dt

= MA

dv/dt is acceleration of centre of mass, MA is the force external. So,

dP

dt

$= F_{ext}$

This above equation is nothing but Newton's second law to a system of particles. If the total external force acting on the system is zero,

$F_{ext} = 0$ then,

dP

dt

$= 0$

This means that P = constant. So whenever the total force acting on the system of a particle is equal to zero then the total linear momentum of the system is constant or conserved. This is nothing but the law of conservation of total linear momentum of a system of particles.

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Conservation of Total Linear Momentum of a System of Particles

Let us take the example of radioactive decay. What is radioactive decay? It is a process where an unstable nucleus splits up in relatively stable nuclei releasing a huge amount of energy.

Suppose there is a parent nucleus which is unstable and it wants to become stable, in order to attain stability it will emit α particle and another daughter nucleus.

This daughter nucleus is much more stable than the parent nucleus. This what radioactive decay is. Now suppose the parent nucleus is at rest and also the mass of the α is ‘ m ‘ and the daughter nucleus is M.

So the mass of the parent nucleus will be m + M. Here everything that is happening is not due to the external force but all that happens is due to the internal force. So here $F_{ext} = 0$, we can say that

$$\frac{dP}{dt} = 0 \Rightarrow P = \text{constant}$$

Solved Questions For You

Q1. Which of the following are practical applications of the law of conservation of linear momentum?

A. When a man jumps out of the boat on the shore, the boat is pushed slightly away from the shore.
B. The person left in the frictionless surface can get away from it by blowing air out of its mouth or by throwing some object in

the direction opposite to the direction in which he wants to move.

C. Recoiling of a gun

D. None of these

Solution: A, B, and C

Q2. Two unequal masses are tied together with a compressed spring. When the cord is burnt with a matchstick releasing the spring; the two masses fly apart with equal :

A. Momentum

B. Acceleration

C. Speed

D. Kinetic energy

Solution: A. Initially, two unequal masses are tied together with a compressed spring. Then the cord is burnt with the matchstick and the spring released due to this the two masses fly apart and acquire velocities in inverse proportional to their masses and hence fly with equal momentum.

Torque and Angular Momentum

While riding a two-wheeler bike, you are able to balance the bike. Even if you increase your speed and move fast, you are still able to maintain the balance. Do you know why? The answer to this is Angular Momentum. Spinning the frisbee or the football are the examples of angular momentum. Let us study this in detail.

Torque

Torque is the rotational analogue of force. It is also termed as the moment of force and denoted by τ. Suppose you have a rod and you apply the force to the rod from upwards. What will happen is, the rod will start moving downwards. Again if you apply an equal and opposite force from downwards. So as both the time equal and opposite forces are applied, the net force is 0 and the rod will definitely not move.

Again take the same rod and apply equal and opposite forces to the rod, but this time apply the force to the two ends of the rod. You will notice that the rod starts rotating. So it is not true that every time you apply equal and opposite force the net force will be zero.

This is true only when the forces are applied in the same force line. So this kind of force in rotational motion is called as torque. So it is not only the force but how and when the force is applied in rotational motion.

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Mathematical Expression of the Torque

Torque is defined as the cross product of the radius vector and the force

$$\tau = r \times F = r\, F\, \text{Sin}\theta$$

- r = position vector of the point on which the force is applied
- F = magnitude of a force
- θ = angle between r and F

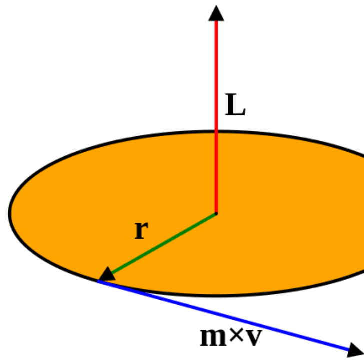
Torque is a vector quantity as it is defined by magnitude as well as direction. Its SI unit is Nm. The dimension of the torque is ML^2T^{-2}

Angular Momentum

Torque and angular momentum are closely related to each other. Angular momentum is the rotational analogue of linear momentum 'p' and is denoted by 'l'. It is a vector product. Angular momentum of the particle is

$$l = r \times p$$

$l = r\, p \sin\theta$, where θ is the angle between r and p.



Relation between Torque and Angular Momentum

As we know, $l = r \times p$, differentiating on both the sides,

$\Rightarrow$

dl

dt

$=$

d

dt

$(r \times p)$

$\Rightarrow$

dl

dt

$=$

dr

dt

$\times p + r \times$

dp

dt

$\Rightarrow$

dl

dt

$= v \times p + r \times$

dp

dt

$\Rightarrow$

dl

dt

$= v \times mv + r \times$

dp

dt

since, $v = 0$

$\Rightarrow$

dl

dt

$= r \times$

dp

dt

$$= r \times F = \tau$$

$\Rightarrow$

dl

dt

$$= \tau$$

So the time rate of change of angular momentum is equal to torque. So finally we can say that rate of change of angular momentum is equal to the torque acting on it. The angular momentum of the system of a particle is denoted by :

$$L = l_1 + l_2 + \ldots..l_n$$

This can be written as $\Sigma = l_i$. Now we know that $l = r \times p$, so

$$l_i = r_i \times p_i$$

The total angular momentum of the system is $\Sigma\, r_i \times p_i$

Torque for System of Particles

We know that,

dl

dt

$= \tau$, a total torque of the system will be the summation of each of the particles.

Σ

dl

dt

$$\tau_i = r_i \times F_i$$

$$\Sigma = r_i \times F_i$$

Torque is external as well as internal.

$$\tau = \tau_{ext} + \tau_{int}$$

$$\tau_{ext} = \Sigma \; r_i \times F_{iext}$$

$$\tau_{in} = \Sigma \; r_i \times F_{iint}$$

So here if internal force becomes zero,

τext =

dl

dt

Hence we can say that time rate of the total angular momentum of the system of particles about a point is equal to the sum of the external torques acting on the system taken about the same point.

Questions For You

Q1. A horizontal flat platform is rotating with uniform angular velocity around the vertical axis passing through its centre. At some instant, a viscous fluid of mass m is dropped at the centre and is allowed to spread out and finally fall. The angular velocity during this period :

A. decreases continuously
B. decrease initially and decreases again
C. remains unaltered
D. increases continuously

Answer: B. The liquid when initially spreads towards the circumference of the platform, its rotational inertia starts increasing as more and more mass is disturbed from the centre. This reduces the angular velocity using conservation of angular momentum. When the fluid finally drops from the platform the rotational inertia decreases which increase the angular velocity.

Q2. Angular momentum is

A. Polar vector
B. Axial vector
C. A scalar
D. None of these

Answer: B. Angular momentum is, $L = r \times m(v)$. L is the axial vector.

Equilibrium of a Rigid Body

A rigid body is a system of many particles. It is not essential that each of the particles of a rigid body behaves in a similar manner like the other particle. Depending on the type of motion every particle behaves in a specific way. This is where the equilibrium of rigid bodies comes into play. How this equilibrium affects the whole system of particles is what we shall learn now.

Equilibrium of Rigid Bodies

Rigid bodies are those bodies in which the distance between particles is constant despite any kind of external force. So while studying the equilibrium of rigid bodies, we mainly aim to define the behavior of these constituting particles in changed conditions of force or torque.Since we are concentrating on the equilibrium of rigid bodies under motion, therefore we need to take both translational and rotational motion into consideration.

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Explanation

Equilibrium is a defined as any point where the total amount of external force or torque is zero. This point may be anywhere near the center of mass. External force in translational motion of the rigid body changes the linear momentum of that body. While the external torque in rotational motion can change the angular momentum of the rigid body.

In the mechanical equilibrium of a rigid body, the linear momentum and angular momentum remain unchanged with time. This implies that the body under the influence of external force neither has a linear acceleration nor an angular acceleration. We, therefore, can say that:

- If the total force on a rigid body is zero then the body shows translational equilibrium as the linear momentum remains unchanged despite the change in time:

- If the total torque on a rigid body is zero then the body shows rotational equilibrium as the angular momentum does not change with time.

Mechanical Equilibrium

When we sum up the above findings of translational and rotational equilibrium we get the following assumptions:

$$F_1+F_2+F_3+F_4+\ldots..F_n= F_i = 0 \quad \text{(For translational equilibrium)}$$

$$\tau_1+\tau_2+\tau_3+\tau_4+\ldots\ldots\tau_n = \tau_i = 0 \text{ (For rotational equilibrium)}$$

These equations are the vector in nature. As scalars the force and torque in their x,y, and z components are:

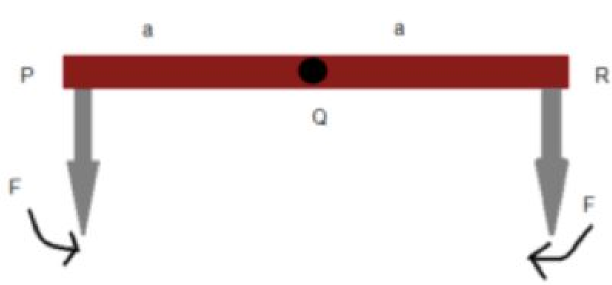
$$F_{ix} = 0\ ,\ \ F_{iy} = 0, \text{ and } F_{iz}= 0 \text{ and } \tau_{ix} = 0\ ,\ \ \tau_{iy} = 0, \text{ and } \tau_{iz}= 0$$

The independent condition of force and torque helps in reaching the rigid bodies to a state of mechanical equilibrium. Generally, the forces acting on the rigid body are coplanar. The three conditions if satisfied, help the rigid body attain equilibrium. The condition of translational equilibrium arises when any of the two components along any perpendicular axis sum up to be zero.

For rotational equilibrium, it is necessary that all the three components result in a zero. Moreover, as the translational equilibrium is a condition that depends on a particle's behaviour, therefore, the vector sum of forces on all the particles must be a zero.

Partial Equilibrium

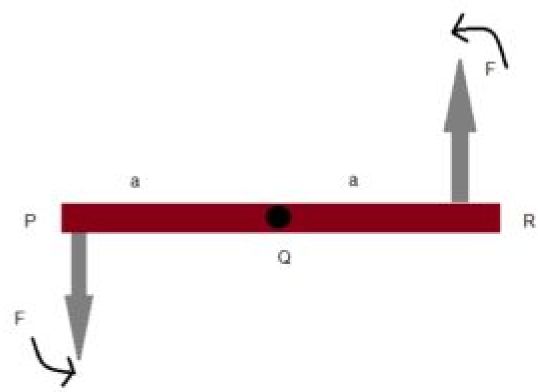
Equilibrium in a rigid body may also be partial in nature. Partial equilibrium of a body is that state where a rigid body shows only one kind of equilibrium. For example, consider the following figure:



The figure shows an instance of rotational equilibrium. Q being the center with sides PQ=QR=a. The forces F at points P and Q are equal in magnitude but opposite in direction.The system is in rotational equilibrium as the net moment on the rod is zero. The translational equilibrium can only be seen if the forces from points P and Q are opposite in perpendicular direction to the rod.

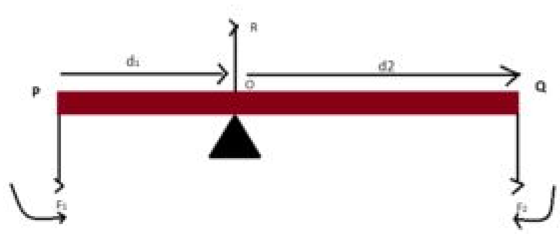
The figure below shows translational equilibrium. In the figure below, the moments from forces at points P and Q are equal. The forces F are not opposite rather they act in the same sense thus causing anti-clockwise rotation of the rod. The total force hence is zero. Here, since the rod without any translation shows rotation. The kinds of force acting on the rod are termed as couple or torque.

A pair of equal and opposite forces with different lines of action that produces rotation of the body is called the torque on that body.



The Principle of Moments

For understanding mechanical equilibrium we need to understand the working of a fulcrum and lever. These pose as the best examples of mechanical equilibrium. A See-saw in parks best explains the principle of lever and fulcrum. See-saw is the lever while the point at which the rod is pivoted is the fulcrum.



The lever here shows mechanical equilibrium. R, the reaction of the support from the fulcrum, is directed opposite the forces, F_1 and F_2 and at $R - F_1 - F_2 = 0$, we see that the rigid body attains translational equilibrium. Rotational equilibrium is attained when $d_1F_1 - d_2F_2 = 0$. For rotational equilibrium, the sum of moments about the fulcrum is zero.

Here, F_1 = load. F_2 = effort needed to lift the load, d_1 = load arm and d_2 = effort arm. At rotational equilibrium, $d_1F_1 = d_2 F_2$ or $F_1/F_2 = d_2/d_1$. The ratio F_1/F_2 is also called the Mechanical advantage. Now, if

$d_2 > d_1$ then mechanical advantage is greater than 1, which means that a small effort can lift the load.

Centre of Gravity

Center of gravity is the point of balance of a rigid body. This situation is the result of the mechanical equilibrium between the two rigid bodies. For example, when you hold a book on the tip of your finger the center at which the book is balanced is called the center of gravity. The mechanical equilibrium between the finger and book has made the balance possible.

The reaction of your fingertip at the center is equal and opposite to Mg (the force of gravity). This balance is an example of translational and rotational equilibrium. The center of gravity of the book is located at the point where the total torque due to force mg is zero. Center of Gravity is, therefore, that point where the total gravitational torque on the rigid body is zero.

Sample Question For You

Q: Consider the following two statements A and B and identify the correct choice:

A) The torques produced by two forces of the couple are opposite to each other

B) The direction of torque is always perpendicular to the plane of rotation of the body

a) A is true and B is false. b) B is true and A is false. c) Both are true. d) A and B are false.

Solution: b) The direction of torque is always perpendicular to the plane of rotation of body as a cross product is in the perpendicular plane to r and F vectors and the torques produced by two forces of the couple are in the same direction to each other.

Angular Momentum in Case of Rotation About a Fixed Axis

Consider yourself rotating around the same axis, with hands stretched. The phenomenon seems simple, but did you ever wonder that with every rotation that you make the Angular momentum changes. Now if you bring your hands closer to the axis of your rotation, what will be the inertia in such case? Let's learn more about system of particles and rotational motion.

Angular Momentum

Earth rotates around its own axis, a ballet dancer with his hands stretched rotates around his own axis! These are some of the most common examples of rotation around a fixed axis. The speed of the ballet dancer changes when his stretched hands are folded inwards. Why does this happen? Well, the reason is the changing angular momentum during the circular motion of the ballet dancer.

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What exactly is angular momentum? Angular momentum can be defined as the vector product of the angular velocity of a particle and its moment of inertia. When a particle of mass m shows linear momentum (p) at a position (r) then the angular momentum with respect to its original point O is defined as the product of linear momentum and the change in position. Here the rotational momentum is taken as l, so,

$$l = r \times p$$

The magnitude of l as a vector shall be,

$$l = rp_1 \sin \theta$$

θ here is the angle between the relative position (r) and linear momentum (p) while p1 is the magnitude of p. We can also write this as:

$$l = rp' \text{ or } r'p$$

r' here is r sin θ and is a distance perpendicular from the point of origin. p' is taken as p sin θ which is the component of p and is in a direction . to the relative position (r). Now at any point where r = 0 or when the linear momentum disappears, angular momentum also becomes a zero. Linear momentum (p) vanishes or becomes a zero when its line of direction passes through the origin or when θ is 0º or 180°.

When calculating the angular momentum for any particle we need to know the relation between the moment of a force and angular momentum. The relation between the two is same as that of force and linear momentum. From l=r × p, when we differentiate it with respect to time we get,

$$dl/\ dt = d\ (r \times p)\ /\ dt$$

Applying the product rule for differentiation,

$$d (r \times p) / dt = dr/dt \times p + dp/dt \times r$$

Now, we already know that velocity is the change in position at some time interval, thence, $dr/dt = v$ and $p = mv$,

$$dr/dt \times p = v \times mv$$

Now since both are parallel vectors, their products shall be a zero (o). Now let's take $dp/dt \times r$,

$$F = dp/dt$$

$$dp/dt \times r = F \times r = \tau$$

This means that, $d (r\times p)/ dt = \tau$. Since $l= r\times p$, therefore,

$$dl / dt = \tau$$

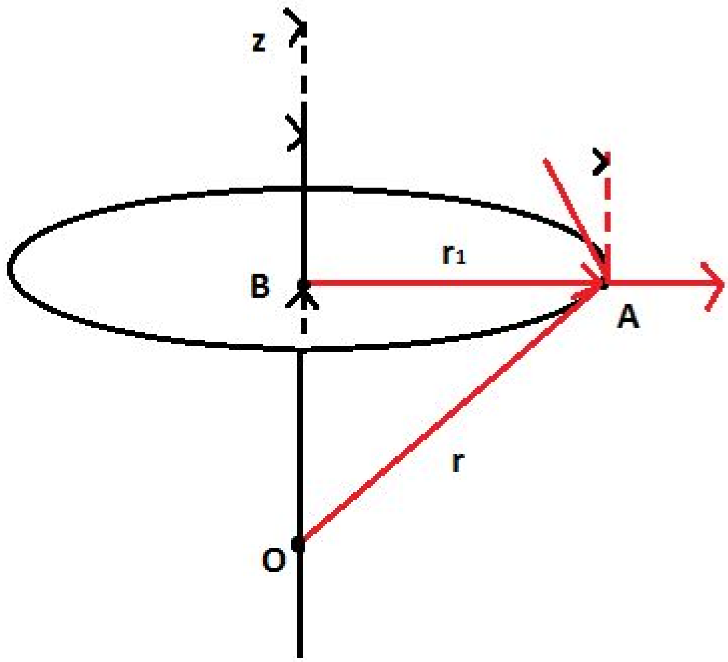
From the above equation, we can say that the rate of change of angular momentum is directly related to the torque on it. Torque (τ) is the external force on the particle.

Angular Momentum for Rotation About a Fixed Axis

From the above discussion we now know that for a system of particles, the time rate of total angular momentum about a point equals the total external torque acting on the system about the same point. Angular momentum is conserved when total external torque is zero. The angular momentum studied here is on a particle on any point.

Here we shall deal with angular momentum about a fixed axis. When we study rotational momentum in reference to a rigid body, we take it as a vector acting on a system of particles. During rotational motion every particle behaves differently, hence we calculate the angular momentum for a system of many particles.

The angular momentum of any particle rotating about a fixed axis depends on the external torque acting on that body. The angular momentum discussed here, is that of a rigid body rotating about a fixed axis. Before we start, let's see the figure below:



General Equation

For total angular momentum(L) we take the following general equation;

$$L = {}_{t=1}N\sum r_t \times p_t$$

As already mentioned that when we calculate the angular momentum, we initially take it on an individual particle and then sum up the contributions of the individual particle. For any particle, l= r×p. From the figure above, r = OA. Using the right angle rule, we take OA = OB+BA. Substituting these values in r we get,

$$l = (OB + BA) \times p = (OB \times p) + (BA \times p)$$

Since, $p = mv$, hence $l = (OB \times mv) + (BA \times mv)$. The linear velocity (v) of the particle at point A is given by:

$$v = \omega r_1$$

r_1 is the length of BA which is the perpendicular distance of point A from the axis of rotation. v is tangential at A to the circular motion in which the particle moves. With the help of the right-hand rule, we know that $BA \times v$, which is parallel to the fixed axis. The unit vector along the fixed axis is k'. From the above equation,

$$BA \times mv = r_1 \times (mv)\, k' = m\, r_1 2\, \omega$$

Likewise, we can say that $OB \times v$ is perpendicular to the fixed axis. Denoting a part of l along fixed axis z as l_z we get,

$$l_z = BA \times mv = m\, r_1 2\, \omega$$

$$l = l_z + OB \times mv$$

We already know that l_z is parallel to the fixed axis while l is perpendicular. Generally, angular momentum l is not along the axis of rotation which means that for any particle l and ω are not impliedly parallel to one another, but for any particle p and v are parallel to each other. For a system of particles, total angular momentum,

$$L = l_t = l_{tz} + OB_t \times m_t v_t$$

We denote L_1 and L_z as the components of L that are perpendicular to the z-axis and along the z-axis respectively; hence

$$L_1 = OB_t \times m_t v_t$$

Here, m_t and v_t are mass and velocity of a t_{th} particle and B_t is the centre of the circle of motion described by the particle t.

$$L_z = l_{tz} = {}_t\sum m_t r_t{}^2 \omega k'$$

$$L_z = I \omega k'$$

According to the definition of Moment of Inertia I,= $m_t r_t{}^2$ which is substituted in the above equation. As already said, in rotational motion we take angular momentum as the sum of individual angular momentums of various particles. Therefore,

$$L = L_z + L_1$$

Angular Momentum for Symmetrical and Asymmetrical Bodies

It has to be noted that, the rigid bodies that we are considering here are symmetric in nature. For symmetric bodies, the axis of rotation is in symmetry with one of their axes. For bodies like these, every particle experiences a velocity *v* which is opposed by another velocity -*v* that is located diametrically opposite in the circle covered during the rotation by the particle. Both the negative and positive gradients of *v* will give L_1 as zero in symmetric bodies. Hence,

$$L=L_z = I\omega k$$

For asymmetric bodies, however, $L \neq L_z$ and L hence do not lie along the rotational axis of the said particle.

Differentiation of Angular Momentum

When we take the equation, $L_z = I\omega k$, we get

$$d(L_z)/dt = k\ d(I\omega)/dt$$

k here is a constant vector, we already know that $dL/dt = \tau$, therefore, the external torque of only those components need to be considered which are along the axis of rotation in a fixed axis. So now we take $\tau = \tau_1 k$.Since the direction of L_z is fixed we assume the $L = L_z + L_1$ therefore for a rotation along a fixed axis we get,

$$dL_z/dt = \tau_1 k \text{ and } dL_1/dT = 0$$

The angular momentum which is perpendicular to the fixed axis is constant for the particle rotating along the same axis. Now since $L_z = I\omega k$, therefore

$$d(I\omega)/dt = \tau_1$$

In a situation where the moment of inertia (I) does not change with time, the equation becomes:

$$d(I\omega)/dt = I\ d\omega/dt = I\alpha$$

Conservation of Angular Momentum

According to the principle of conservation of angular momentum, in the absence of any external torque, the angular momentum remains constant, no matter what the interaction and transformation within the system of rotation. This, therefore, means that if there is no external torque along the axis of rotation then $I\omega$ is constant. Here we neglect the friction acting during the mechanism of rotation.

With the help of Angular momentum in the rotation along the fixed axis, we can now understand how various acrobats and skaters manage their angular speed during their actions.

You must have noticed yourself that when on a rotating chair, the moment you stretch your hands the speed of rotation decreases while when your hands are brought back closer to your body, the same speed increases. This is due to the changing external torque and angular momentum. A principle that dictates the action of acrobats, skaters, and dancers during various performances, angular momentum helps them master the art with excellence!

Solved Examples for You

Question: Which of the following statements about angular momentum is correct? It is…

A. the moment of inertia

B. a scalar vector

C. directly proportional to the moment of inertia

D. none of the above

Solution: C. L= Iω, angular momentum is a vector quantity that is directly proportional to the moment of Inertia.

Dynamics of Rotational Motion about a Fixed Axis

We already know that a rigid body may follow a translational or rotational motion. There may be objects which follow both, hence while studying the rigid body dynamics of such objects we compare the kinematics of both the motions. The page below deals with the rigid body dynamics that follows rotational motion about a fixed axis.

Rigid Body Dynamics of Rotational Motion

When in motion, a rigid body is believed to be a system of particles. Each of its particles follows a path depending on the kind of motion it follows. In a translational motion, all the particles move and behave in a similar manner. But in rotational motion, the rigid body dynamics indicate a different behaviour.

All the particles behave differently. Since rotation here is about a fixed axis, every particle constituting the rigid body behaves to be rotating around a fixed axis. As the distance from the axis increases the velocity of the particle increases.

A particle in rotational motion moves with an angular velocity. Moment of inertia and torque for the rotational motion are like mass and force in translational motion. These analogues for both the motions give us the idea of a particle's behaviour while in motion. While describing the rigid body dynamics during rotational motion about a fixed axis we intend to highlight the relation between the analogues of both the motions.

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Let us learn in detail about Rotational Dynamic.

Components of Interest in Rotational Motion

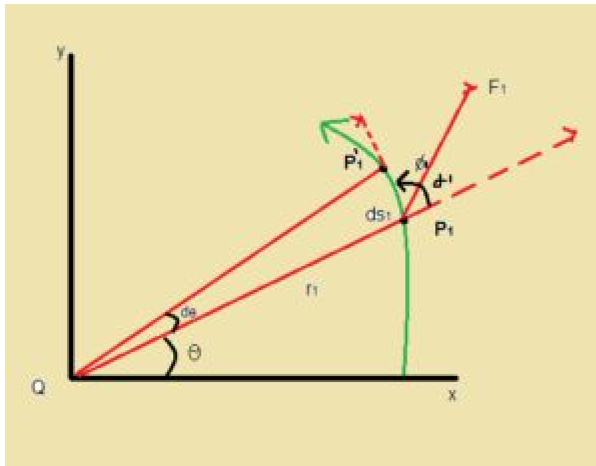
In the rotational motion of a particle about a fixed axis, we take into consideration only those components of torques that are along the direction of the fixed axis. Since these are the components that cause the body to rotate, it is necessary that every component of torque be to lie in a plane perpendicular to the axis of rotation.

The axis will turn from its position if the component of torque is perpendicular to the axis. The fixed axis of rotation is maintained only when the external torque is constrained by necessary forces. This is the reason we do not consider the perpendicular component of torques. We, therefore consider only the forces that lie in the plane perpendicular to the axis.

All the forces that are parallel to the rotational axis will apply torque perpendicular to the axis, hence should be ignored. Another thing to be considered are those components of position vectors that are perpendicular to the axis. Any component of position vector that is along the axis will give a torque perpendicular to the axis and hence can be ignored.

Graphical Representation of a Rigid Body Dynamics

For understanding the dynamics of a rigid body during rotational motion around a fixed axis we need to study the graph below:



The graph above represents work done by a torque acting on a particle that is rotating about a fixed axis. The particle is moving in a circular path with center Q on the axis. $P_1P'_1$ is the arc of displacement ds_1. The graph in the figure shows the rotational motion of a rigid body across a fixed axis.

As already mentioned that while studying rigid body dynamics of rotational motion we consider only those forces that lie in planes perpendicular to the axis of rotation. F1 is the same force acting on particle P1 and lies in a plane perpendicular to the axis.This plane can be called as x'-y' plane and r1 is the radius of the circular path followed by particle P1.

Now from the figure, we know that $QP_1 = r_1$ and particle P1 moves to position P' in time Δt. The displacement of the particle here is ds_1. The magnitude of $ds_1 = r_1 d\theta$. Here, $d\theta$ is the angular displacement of the particle and is equal to $\angle P_1QP'_1$. The work done by the force on particle = $dW_1 = F_1 ds_1 \cos ø_1 = F_1(r_1 d\theta) \sin\alpha_1$.

$ø_1$ is the angle between the tangent at P_1 and the force F_1 and α_1 is the angle between radius vector and F_1.

Torque

In the figure, we notice that the radius vector is 90° $(ø_1 + \alpha_1)$. The torque due to force F_1 around the origin is a product of radius vector and F1. Here we should remember that any particle on the axis is excluded from our analysis. So the effective torque in that case arising due to force F1 is signified by τ.

τ= QP × F_1. Effective torque τ is directed along the axis of rotation with a magnitude of $r_1F_1\sin\alpha$. This brings us to the conclusion that work done = τ_1 dθ.

Workdone by Multiple Forces

The above graph represents the force and work done by the same force on one particle. Now let's consider the case of more than one forces acting on the rigid body. In a system of particles with more than one force acting on the system, the work done by each of them is added, this gives the total work done by the body. The magnitude of torques due to various forces is denoted as τ1, τ2,τ3…… etc. So, dW = (τ1+ τ2+τ3 +……) dθ.

The angular displacement (dθ) for all the particles is same here. All the torques under our consideration are parallel to the fixed axis and the magnitude of the total external force is just the sum of individual torques by various particles. This gives us the equation: dW = τ dθ. This represents the work done by the total torque that acts on the rigid body rotating about a fixed axis.

Relation Between Rotational and Translational Motion

The workdone (dW) by external torque during rotational motion = $\tau\, d\theta$. The workdone (dW) by the external force during translational motion = Fds. Here, ds is the linear displacement and F is the external force. Dividing both by dt we get: $P = dW/dt = \tau\, d\theta/dt = \tau\omega$ or $P = \tau\omega$. Here, P is the instantaneous power. Instantaneous power (P) for linear motion = Fv.

Do you notice any similarity? We know that a perfectly rigid body lacks internal motion hence the work done by external torque increases the kinetic energy of the body. Now, to find the rate of increase in kinetic energy we equate the equations as: $d/dt\,[I\omega 2]/ 2 = I\,(2\omega)/2\,[d\omega/dt]$. From the equation, we assume that moment of inertia is constant and does not change with time.

This also means that the mass of the body also does not change. Now since the axis is also fixed, its position with respect to the body also doesn't change so: using $\alpha = d\omega/dt$. $d/dt\,[I\omega 2]/ 2 = I\omega\alpha$. Therefore the rate of work done and rate of increase in kinetic energy can be equated as $\tau\omega = I\omega\alpha$ or $\tau = I\alpha$. Now, this equation is similar to Newton's Second law for Linear motion, $F = ma$.

Therefore from the relation of work done and kinetic energy we come to a conclusion that Newton's second law of linear motion is applicable to rigid bodies undergoing rotational motion. So, Newton's second law of rotational motion states that the angular acceleration during rotational motion of a rigid body is directly proportional to the applied torque and inversely proportional to the moment of Inertia of that body.

Sample Question For You

Q: A train engine that weighs 5000N stops at the exact center of a bridge of weighs 75,000N and has two equally spaced pillars to support the bridge. Find out the sum of the torques?

A) 75000 N B) 40000 N C) 2500 N D) 0 N

Solution:D) The bridge + train engine system is in equilibrium. Therefore, the net torque about every point must be zero. Hence the correct option is 0 N.

Kinematics of Rotational Motion about a Fixed Point

Suppose there is a motorcycle riding on a road. It is observed that when the acceleration is more the wheels of the bike spins more and rotates through many revolutions. This only happens when the wheels have angular acceleration for a long time. Let us study more about angular acceleration in detail.

Kinematics of Rotational Motion about a Fixed Point

We all know that rotational motion and translational motion are analogous to each other. In rotational motion, the angular velocity is ω which is analogous to the linear velocity v in the transitional motion. Let us discuss further the kinematics of rotational motion about a fixed point.

The kinematic quantities in rotational motion like the angular displacement θ, angular velocity ω and angular acceleration α respectively corresponds to kinematic quantities in linear motion like displacement x, velocity v and acceleration a.

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Angular Acceleration

The rate of change of angular velocity is the angular acceleration.

$\alpha =$

dω

dt

(rad/sec²)

Now let us consider a particle P on a rotating object. The object undergoes a rotation motion at the fixed point. The angular displacement of a particle P is θ. Hence in time t = 0, the angular displacement of the particle P is 0. So we can say that in time t, its angular displacement will be equal to θ.

Angular Velocity

Angular velocity is the time rate of change of angular displacement. We can write it as,

ω =

dθ

dt

As we know that the rotational motion here is fixed, so there is no need to change the angular velocity. We know angular acceleration is α =

dω

dt

. So the kinematics equations of linear motion with uniform acceleration is,

$$v = v_0 + at$$

$x = x_0 + v_0t +$

1

2

$$at^2$$

$$v^2 = v_0^2 + 2ax$$

Where x_0 is the initial displacement and v_0 is the initial velocity of the particle. Here initial means t = 0. Now, this equation corresponds to the kinematics equation of the rotational motion.

Kinematics Equations for Rotational Motion with Uniform Angular Acceleration

$$\omega = \omega_0 + \alpha t$$

$\theta = \theta_0 + \omega_0 t +$

1

2

$$\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Where θ_0 is the initial angular displacement of the rotating body and ω_0 is the initial angular velocity of the particle of the body.

Solved Examples For You

Q1. A wheel rotating with uniform angular acceleration covers 50 rev in the first five seconds after the start. If the angular acceleration at the end of five seconds is x π rad/s^2 find the value of x.

A. 4

B. 8

C. 6

D. 10

Solution: B

$$\theta = \frac{1}{2}\alpha t^2$$

$$\alpha = \frac{2\theta}{t^2} = \frac{2(50)(2\pi)}{5^2} = 8\pi \text{ rad/s}^2 = 25.14 \text{ rad/s}^2$$

comparing with α, $x = 8$ rad/s^2

Q2. Starting from rest, a fan takes five seconds to attain the maximum speed of 400 rpm. Assume constant acceleration, find the time taken by the fan attaining half the maximum speed.

A. 11 s

B. 2.0 s

C. 2.5 s

D. 2.0 s

Solution: D. The maximum angular velocity is given by,

$w_m = 400\text{rpm} = 400 \times$

2π

60

$=$

40

3

rad/sec

Initial angular velocity is $w_m = 0$

So angular accleration α = \(\frac{w_m – ω – 0}{t} \) =

40/3–0

5

=

8π

3

rad/sec²

Now $\omega = w_0 + at$ we get $\omega_{m/2} = 0 + at$ =

40π/3

2(8π/3)

= 2.5s

Q3. Identify the direction of the angular velocity vector for the second hand of a clock going from 0 to 60 seconds?

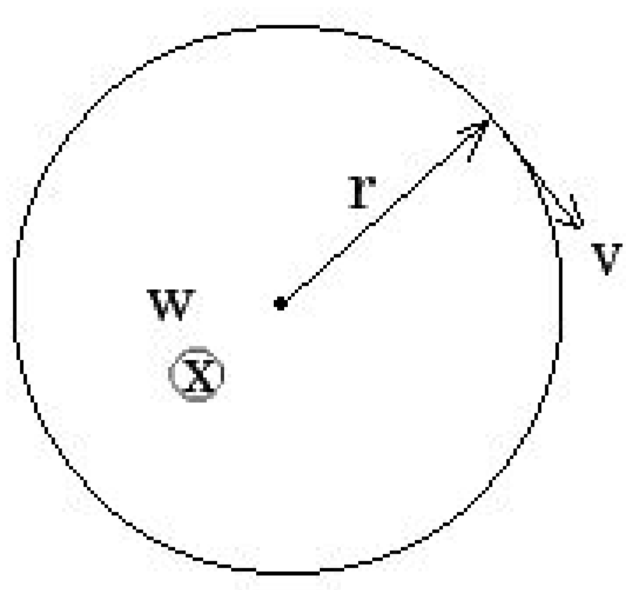
A. outward from clock face

B. inward towards the clock face

C. upward

D. dwonward

Solution : B



Angular velocity = $\omega \times r$. The second hand of the clock rotates in clockwise direction. From the above figure, the direction of angular velocity is into the plane of the page that is inward towards the clock face.