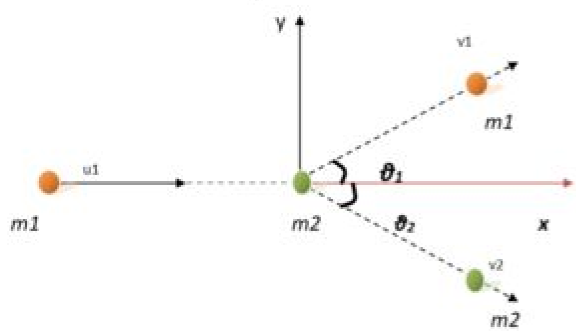
Collisions

You might have seen two billiard balls colliding with each other in the course of the game. This forceful coming together of two separate bodies is called collision. What happens after collisions? Can we determine the velocity or the trajectory of the colliding bodies? Let us find out!

What is a Collision?

Collision means two objects coming into contact with each other for a very short period. In other words, collision is a reciprocative interaction between two masses for a very short interval wherein the momentum and energy of the colliding masses changes. While playing carroms, you might have noticed the effect of a striker on coins when they both collide.

Collision involves two masses m_1 and m_2. The v_{1i} is the speed of particle m_1, where the subscript *'i'* implies initial. The particle with mass m_2 is at rest. In this case, the object with mass m_1 collides with the stationary object of mass m_2.



As a result of this collision the masses m1 and m2 move in different directions.

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Types Of Collision

Generally, the law of conservation of momentum holds true in the collision of two masses but there may be some collisions in which Kinetic Energy is not conserved. Depending on the energy conservation, conservation may be of two types:

- Elastic Collision: In the elastic collision total momentum, the total energy and the total kinetic energy are conserved. However, the total mechanical energy is not converted into any other energy form as the forces involved in the short interaction are conserved in nature. Consider from the above graph two masses, m_1 and m_2 moving with speed u_1 and u_2. The speed after the collision of these masses is v_1 *and* v_2. The law of conservation of momentum will give:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

The conservation of Kinetic Energy says:

$$1/2\ m_1u_1^2 + 1/2\ m_2u_2^2 = 1/2\ m_1v_1^2 + 1/2\ m_2v_2^2$$

- Inelastic Collision: In the inelastic collision, the objects stick to each other or move in the same direction. The total kinetic energy in this form of collision is not conserved but the total momentum and energy are conserved. During this kind of collision, the energy is transformed into other energy forms like heat and light. Since during the phenomenon the two masses follow the law of conservation of momentum and move in the same direction with same the same speed v we have:

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$v = (m_1u_1 + m_2u_2)/(m_1 + m_2)$$

- The kinetic energy of the masses before the collision is : $K.E_1 = 1/2\ m_1u_1^2 + 1/2\ m_2u_2^2$
- While kinetic energy after the collision is: $K.E_2 = 1/2\ (m_1 + m_2)\ v^2$
- But according to the law of conservation of energy: $1/2\ m_1u_1^2 + 1/2\ m_2u_2^2 = 1/2\ (m_1 + m_2)\ v^2 + Q$
- 'Q' here is the change in energy that results in the production of heat or sound.

The Coefficient of Restitution

The coefficient of restitution is the ratio between the relative velocity of colliding masses before interaction to the relative velocity of the masses after the collision. Represented by 'e', the coefficient of restitution depends on the material of the colliding masses. For elastic collisions, e = 1 while for inelastic collisions,e = 0. The value of e > 0 or e < 1 in all other kinds of forceful interactions.

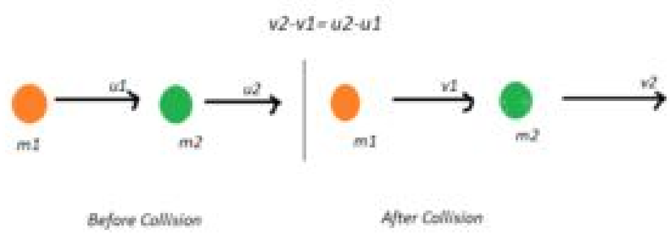
One Dimensional Collision

One dimensional sudden interaction of masses is that collision in which both the initial and final velocities of the masses lie in one line. All the variables of motion are contained in a single dimension.

Elastic One Dimensional Collision

As already discussed in the elastic collisions the internal kinetic energy is conserved so is the momentum. Elastic collisions can be achieved only with particles like microscopic particles like electrons, protons or neutrons.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$



Since the kinetic energy is conserved in the elastic collision we have:

$$1/2\ m_1u_1^2 + 1/2\ m_2u_2^2 = 1/2\ m_1v_1^2 + 1/2\ m_2v_2^2$$

This gives us : $m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2$ (Factoring out 1/2)

Rearranging we get: $m_1u_{21} - m_1v_{21} = m_2v_{22} - m_2u_{22}$

Therefore, $m_1(u_{21} - v_{21}) = m_2(v_{22} - u_{22})$

Which if elaborated become $m_1(u_1+v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2)$

Using the conservation of momentum equation: $m1u1 + m2u2 = m1v1 + m2v2$

We regroup it with same masses: $m1u1 - m1v1 = m2v2 - m2u2$

Hence, $m1(u1 - v1) = m2(v2 - u2)$

Now dividing the two equations:

$m_1(u_1+v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2) / m_1(u_1 - v_1) = m_2(v_2 - u_2)$

We get: $u_1+v_1 = v_2 + u_2$

Now, $v_1 = v_2 + u_2 - u_1$

When we use this value of v_1 in equation of conservation momentum we get :

$$v_2 = [2\, m_1 u_1 + u_2 (m_2 - m_1)] / (m_1 + m_2)$$

Now using the value of $v2$ in equation $v_1 = v_2 + u_2 - u_1$

$$v_1 = [2\, m_1 u_1 + u_2 (m_2 - m_1)] / (m_1 + m_2) + u_2 - u_1$$

$$v_1 = [2\, m_1 u_1 + u_2 (m_2 - m_1) + u_2 (m_1 + m_2) - u_1 (m_1 + m_2)] / (m_1 + m_2)$$

We finally get:

$$v_1 = [2 m_2 u_2 + u_1 (m_1 - m_2)] / (m_1 + m_2)$$

When masses of both the bodies are equal then generally after collision, these masses exchange their velocities.

$$v1 = u2 \quad and\ v2 = u1$$

This means that incourse of collision between objects of same masses, if the second mass is at rest and the firstmass collides with it then after collision the first mass comes to rest and the second mass moves with the speed equal to first mass. Therefore in such case, $v_1 = 0$ and $v_2 = u_1$. In case if $m_1 < m_2$ then, $v_1 = -u_1$ and $v_2 = 0$

This means that the lighter body will bombard back with its own velocity, while the heavier mass will remain static. However, if $m_1 > m_2$ then $v_1 = u_1$ and $v_2 = 2u_1$

Inelastic One Dimensional Collision

In inelastic one dimensional collision, the colliding masses stick together and move in the same direction at same speeds. The momentum is conserved and Kinetic energy is changed to different forms of energies. For inelastic collisions the equation for conservation of momentum is :

$$m_1u_1 + m_2u_2 = (m_1 + m_2)\, v$$

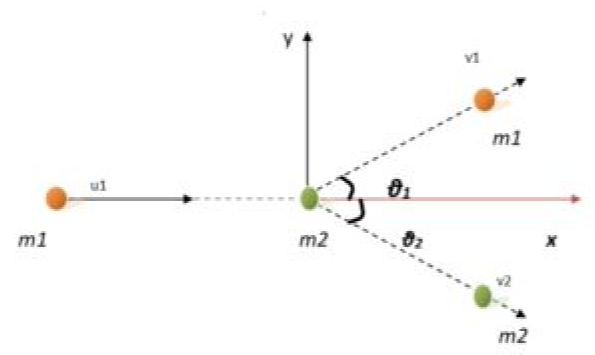
Since both the objects stick, we take final velocity after the collision as *v*. Now *v* shall be:

$$= m_1u_1 + m_2u_2 / m_1 + m_2$$

The kinetic energy lost during the phenomenon shall be:

$$E = 1/2\, m_1u_2^2 - 1/2\, (m_1 + m_2)\, v^2$$

Collision in Two Dimensions



The above figure signifies collision in two dimensions, where the masses move in different directions after colliding. Here the moving mass m_1 collides with stationary mass m_2. The linear momentum is conserved in the two-dimensional interaction of masses. In this case, we see the masses moving in x,y planes. The x and y component equations are:

$$m_1u_1 = m_1u_2\cos\theta_1 + m_2v_2\cos\theta_2$$

$$0 = m_1u_2\sin\theta_1 - m_2v_2\sin\theta_2$$

For spherical objects that have smooth surfaces, the collision takes place only when the objects touch with each other. This is what happens in the games of marbles, carom, and billiards.

Solved Examples For You

Q: A sphere of mass m, moving with a speed v, strikes a wall elastically at an angle of incidence θ. If the speed of the sphere before and after the collision is the same and the angle of incidence and velocity normally towards the wall the angle of rebound is equal to the angle of incidence and velocity normally towards the wall is taken as negative then, the change in the momentum parallel to the wall is:

A) mv cosθ B) 2mv cosθ C) -2mv cosθ D) zero

Solution: D) The mass of the object remains constant. To see the change in the momentum, we need to see the forces applied due to the collision. The only force applied to the sphere of mass 'm' is the normal reaction force due to the wall. This force has no parallel component along the wall. Therefore, the change in momentum in a direction parallel to the wall is zero.

The Concept of Potential Energy

In earlier classes, we defined the potential energy of an object as the energy by virtue of its height. However, here we shall see that this energy can also be a result of an objects configuration or its relative position inside a field. The concept of potential energy is basic to many branches of physics, let's learn more!

Potential Energy

Potential energy is basically that form of energy that's stored in an object due to its position in comparison to a certain zero position. For example, a rock on the top of a hill has a stored energy in it. This energy is relative to its peak position that takes its form when thrown from that height. Similarly, huge ball in the demolition machine has energy stored in it when held at an elevation.

In a nutshell, every object situated at some height, with a possibility to come back on the ground has energy stored in it. This energy is called so because it has the potential to be converted into some other form of energy. This form of energy can be converted into Kinetic energy, Mechanical energy heat and light energy etc.

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Some of the other examples of this form of energy around us

- Electricity produced from water turbines in dams is the result of energy stored in the water of reservoirs.
- The stretched bow is also a good example of potential energy. This energy helps the arrow to move forward with greater velocity.

Kinds of Potential Energy

The potential energy is a function of some field. Whenever there is a field of a force of some kind, any object that interacts with the force gains energy. This energy comes as a result of the interaction of the object with the force or the field. We call this the potential energy.

For example, around a mass, we have the gravitational field. Any mass in that field will experience a force and thus will have an energy associated with it. This is the gravitational potential energy. Similarly, we see the following kinds of potential energy.

Elastic Potential Energy

As the name signifies, every object that behaves like an elastic or spring is a source of elastic potential energy. The best examples are rubber bands, springs etc. These elastic objects follow the Hooke's

Law. The stretching and compressing of elastic items lead to storage of energy in the form of Elastic Potential energy.

Compression of a spring requires force, the more the compression the greater is the force. This implies that amount of force is directly relative to the amount of compression or stretch(x). The constant of proportionality here can be termed as spring constant (k). Therefore, $F = k \times x$.

Thus the force at equilibrium position is zero because at this position the spring is neither stretched nor compressed. The following equation is used when the spring is not in an equilibrium position: $F=1/2\ k \times x$. The energy because of the work that this elastic force does is given by $W = 1/2\ (kx2)$; where the letters have their usual meaning.

Gravitational Potential Energy

We know that Earth has a gravitational force that pulls objects towards the surface. This constant attraction between Earth and the object leads to Gravitation Potential energy. This form of energy depends on the mass and height of the object. Gravitational potential energy is directly related to both mass and height.

The greater the mass the greater will be the gravitational potential energy. Similar is the case with height, an object at a higher elevation will have greater gravitational energy stored in it.

An Instance of the Gravitational Potential Energy

Suppose an object of mass M is lying at point A on the surface of the earth. g here is the acceleration due to gravity on Earth's surface and the force of attraction towards the center of the Earth is M*g, which equals the weight of the object. Now, If we have to move the object to B, from point A then the force that equals M*g has to be applied in the upward direction.

The amount of work done by the object at height h shall be M*g*h.This is also the gravitational potential energy stored in the object at height h. Therefore, $V = M \times g \times h$ (g here is constant, 9.8N/kg). It is essential to know that the gravitational potential energy solely depends on the displacement of the object from an initial height to the final height.

Now, if the object moves upwards to height (h) then the gravitational force (F) acting on the object is negative of P.E. So here when an object moves away from the surface we get, F = -d/dh (V) = -mg

The negative sign here denotes that as the object is moving upwards, the gravitational force is pulling it downwards. The kinematic relation here comes into play and is denoted by the change in speed when the object falls back to the ground. Here the speed of this falling object increases. Hence, $v^2 = 2gh$ or $(1/2) mv^2 = mgh$

Energy as a Negative Constraint

This denotes that the potential energy of the object of mass m at a height h converts into kinetic energy as soon as it falls on the ground. As the force acting on an object situated at a height is against the gravitational force from earth, the potential energy in that object behaves as a negative constraint.

But this energy by virtue of heigh becomes a positive constraint as soon as the object reverts to the ground. Potential energy (V) further, for a force F (x) is:

$\int F(x)\,dx = -\int V dV = V_i - V_f$ [here $F(x) = -dV/dx$]. V_i is the initial value of the potential or the value of the potential at the initial position and V_f is its value at the final position. The distance between the two points is irrelevant here as gravity depends only on the initial and final positions of the object.

The potential energy stored in an object has dimensions ML^2T^{-2} and the unit here is Joule or J. This is similar to Kinetic energy or work and the change in Potential energy with respect to the conservative force acting on it is the negative of the total work done by that force. Therefore, $\Delta V = -F(x)\Delta x$

Chemical Potential Energy

The attractive force between two atoms forms a chemical bond. This bond between atoms and molecules results in storage of energy in the form of Chemical potential energy.The potential energy during chemical bonds and reactions converted into heat and light. The normal batteries and cells are the best examples of Chemical Potential energy.

Solved Example For You

Q: A body is falling from a height h. After it has fallen a height h/2, it will possess

a. only potential energy
b. only kinetic energy
c. half potential and half kinetic energy
d. more kinetic and less potential energy

Solution: (c) At height h the K.E of the object is 0 while P.E is mgh. At height h/2 the P.E becomes mgh/2 while the other half of P.E is converted into K.E due to the virtue of its motion. So mgh – mgh/2 = mgh/2. Hence, the body has half its energy in the form of potential energy and the other half is in the form of kinetic energy

Conservation of Mechanical Energy

Have you ever wondered how an automatic mechanical watch works? At the center of its complicated motion lies one of the most basic principles of classical physics: The law of conservation of mechanical energy. Let's delve into the principle:

Mechanical Energy

It is the capacity of an object to do work by the virtue of its motion or configuration (position). Mechanical Energy is the sum of following two energy terms:

- Kinetic Energy. It is the ability of an object to do work by the virtue of its motion. For example, the kinetic energy of Wind has the capacity to rotate the blades of a windmill and hence produce electricity. Kinetic energy is expressed as,where, K is the kinetic energy of the object in joules (J), m is the mass of the object in kilograms and v is the velocity of the object:

$$K = \frac{1}{2}mv^2$$

- Potential energy. It is the ability of an object to do work by the virtue of its configuration or position. For example, a compressed spring can do work when released. For the purpose of this article, we will focus on the potential energy of an object by the virtue of its position with respect to the earth's gravity. Potential energy can be expressed as:

$$V = \mathrm{mg}\,h$$

Here, V is the potential energy of the object in joules (J), m is the mass of the object in kilograms, g is the gravitational constant of the earth (9.8 m/s^2), and h is the height of the object from earth's surface. Now, we know that the acceleration of an object under the influence of earth's gravitational force will vary according to its distance from the earth's centre of gravity.

But, the surface heights are so minuscule when compared to the earth's radius, that, for all practical purposes, g is taken to be a constant.

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Conservation of Mechanical Energy

The sum total of an object's kinetic and potential energy at any given point in time is its total mechanical energy. The law of conservation of energy says "Energy can neither be created nor be destroyed."

So, it means, that, under a conservative force, the sum total of an object's kinetic and potential energies remains constant. Before we dwell on this subject further, let us concentrate on the nature of a conservative force.

Conservative Force

A conservative force has following characteristics:

- A conservative force is derived from a scalar quantity. For example, the force causing displacement or reducing the rate of displacement in a single dimension without any friction involved in the motion.

- The work done by a conservative force depends on the end points of the motion. For example, if W is the work done, $K_{(f)}$ is the kinetic energy of the object at final position and $K_{(i)}$ is the kinetic energy of the object at the initial position:

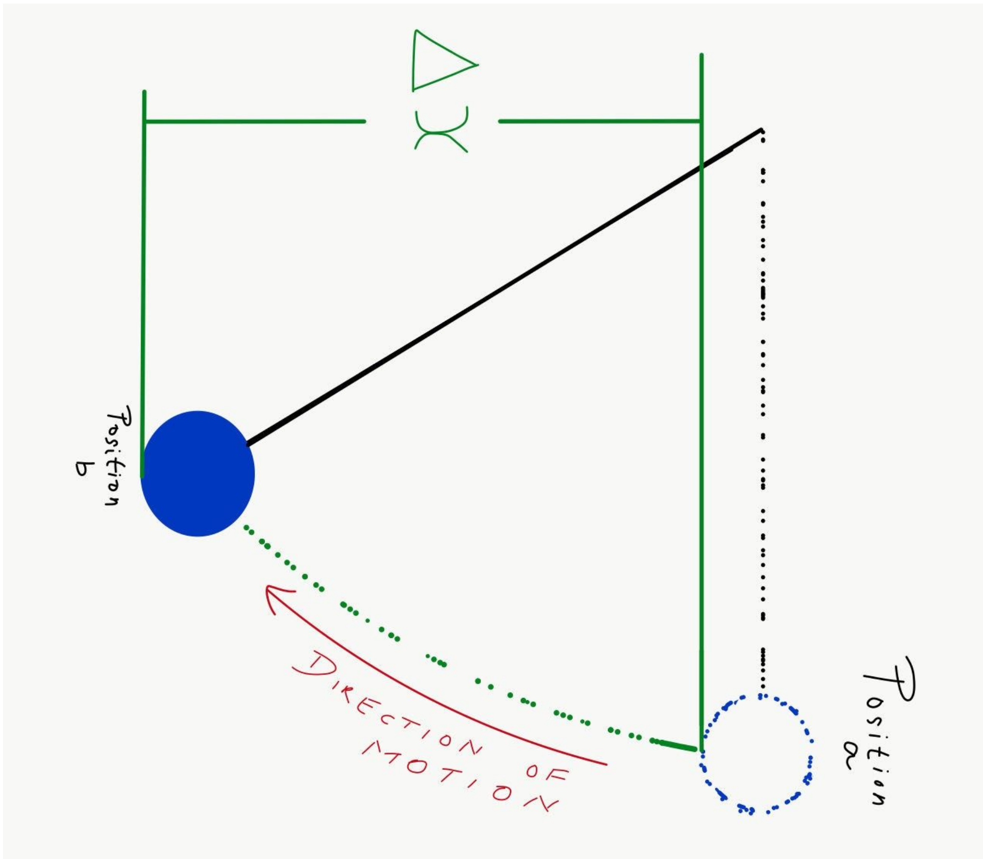
$$W = K_f - K_i$$

- Work done by a conservative force in a closed path is zero. Here, W is the work done, F is the conservative force and d is the displacement vector. In case of a closed loop, the displacement is zero. Hence, the work done by the conservative force F is zero regardless of its magnitude.

$$W = \vec{F} . \vec{d}$$

Proof of Conservation of Mechanical energy

Let us consider the following illustration:



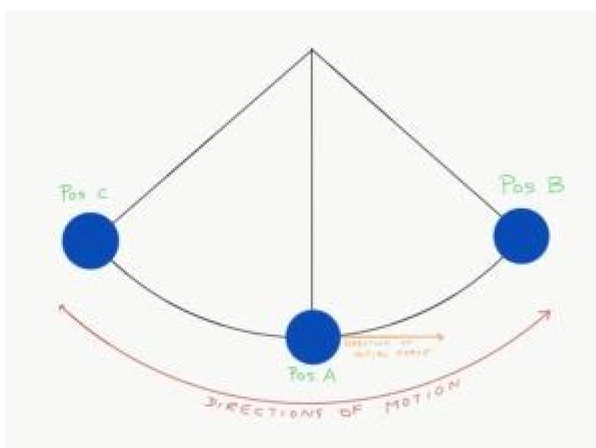
Here, Δx is the displacement of the object under the conservative force F. By applying the work-energy theorem, we have: $\Delta K = F(x)\,\Delta x$. Since the force is conservative, the change in potential Energy can be defined as $\Delta V = -F(x)\,\Delta x$. Hence,

$$\Delta K + \Delta V = 0 \text{ or } \Delta(K + V) = 0$$

Therefore for every displacement of Δx, the difference between the sums of an object’s kinetic and potential energy is zero. In other words, the sum of an object’s kinetic and potential energies is constant under a conservative force. Hence, the conservation of mechanical energy is proved.

Case Study: Simple Pendulum

The pendulum is a very good example of conservation of mechanical energy. Following illustration will help us understand the pendulum motion:



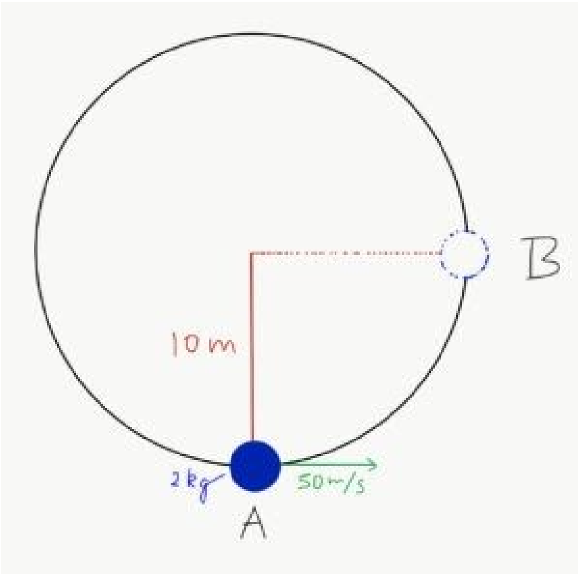
- At position A, Potential energy is zero and the kinetic energy is at maximum.

- When the object travels from position A to B, it’s kinetic energy reduces and potential energy increases.
- At position B, the object stops momentarily. At this position, the object’s kinetic energy becomes zero and its potential energy reaches the maximum. The law of conservation of mechanical energy comes into play here. The object’s entire kinetic energy at position A has been converted to potential energy at position B.
- Now, the object retraces its path, this time from position B to position A. Back at position A, the object’s kinetic energy has been restored to its initial level. Object’s Potential energy is zero.
- Now, the object travels the exact same path as AB, but in reverse direction of AC.
- This process repeats itself infinitely because the mechanical energy of the object remains constant.

This property of mechanical energy has been harnessed by watchmakers for centuries. Of course, in the real world, one has to account the other forces like friction and electromagnetic fields. Hence, no mechanical watch can run perpetually. But, if you get a precise mechanical watch like Rolex, you can expect long power reserves!

Solved Examples For You

Q: A mass of 2kg is suspended by a light string of length 10m. It is imparted a horizontal velocity of 50m/s. Calculate the speed of the said mass at point B.



Solution:

Potential energy at point A, V(A) = mgh(A)

Kinetic energy at point A, K(A) = $(mv^2)/2 = (2 \times 2500)/2 = 2500J$

Hence, total mechanical energy at point A, $K(A) + V(A) = [2500 + V(A)]J$

Potential energy at point B, $V(B) = mg\,h(B) = mgh\,(A+10) = mg\,h(A) + 2 \times 9.8 \times 10 = [V(A) + 196]J$

Kinetic energy at point B, $K(B) = (mv^2)/2$

Hence, total mechanical energy at point B, $K(B) + V(B) = [K(B) + V(A) + 196]J$

By applying the law of conservation of energy,

$V(A) + K(A) = V(B) + K(B)$

Therefore, $V(A) + 2500 = K(B) + V(A) + 196$

or $K(B) = 2500 – 196$

Which gives: $(mv^2)/2 = 2304$

$(2\times v^2)/2 = 2304$

v = [2304]½

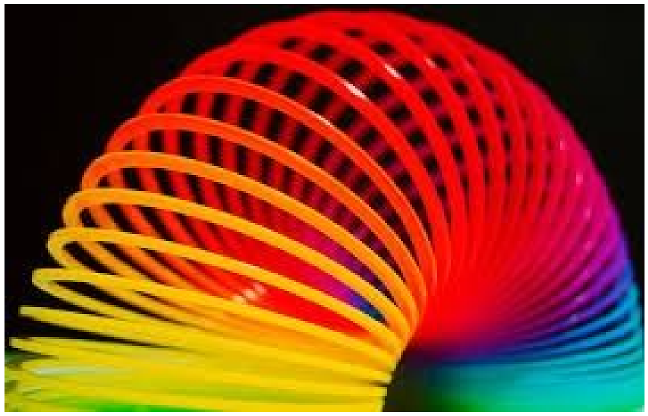
Therefore, velocity of the mass at point B = 48m/s

Potential Energy of a Spring

Have you ever noticed that the spring regains its normal shape despite the force you put while compressing or stretching it? Why do you need to exert extra stress to change a springs position? The secret is the stored Spring potential energy. The physics behind the work, energy, and force of elastic substances like springs! Let's understand the Spring potential energy.

Spring potential energy

When you compress or stretch a spring, as soon as the stress is relieved, the spring attains its normal shape instantly. Its Elastic potential energy helps it do so. Generally, these elastic substances follow the Hooke's law.



Hooke's Law

Before unveiling the mechanism of spring potential energy, we need to understand the Hook's Law. According to this Law, the force needed to change the shape of spring is proportional to the displacement of the spring. The displacement referred here is how far the spring is compressed or stretched from its normal shape. Mathematically, Hook's Law can be summarised as $F = -kx$.

Here 'k' is the spring constant and 'x' is the displacement. This spring constant being unique for different springs depends on various factors like the material of the spring and the thickness of the coiled wire used in the spring. Hooke's law is frequently represented in the negative form since the force is a restoring force, but the positive version of the law is also a valid representation.

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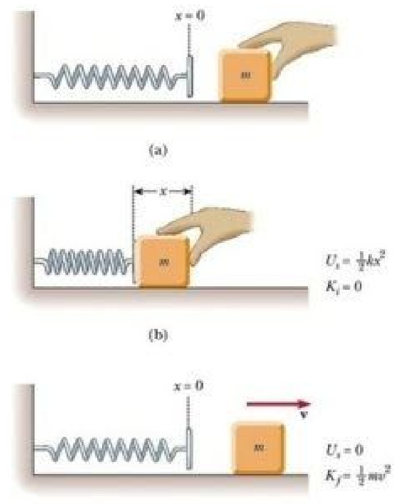
Spring potential energy

To find the Spring potential energy, we need to use the Hooke's law. Since the potential energy is equal to the work done by a spring and work, in turn, is the product of force and distance, we get our force from Hooke's law. Distance here is the displacement in the position of the spring.



In the figure, x is the displacement from the equilibrium position. When we pull the spring to a displacement of x as shown in the figure, the work done by the spring is :

$$W = \int_0^{x_m} F dx = -\int kx\, dx = -k(x_m)^2/2$$

The work done by pulling force F_p is :

$$F_p = k\,(x_m)^2 / 2$$

The work done by the pulling force F_p is in positive as it has overcome the force of spring. Therefore,

$$W = k\, x_m^2 / 2$$

When displacement is less than 0, the work done by the springs force is

$$W_s = -kx_c^2 / 2$$

and the work done by the external force F is $= + kx_c^2/2$. In the process of displacement of the object from initial displacement x_i to final displacement, x_f the work done is,

$$W_s = -\int_{x_i}^{x_f} kx\,dx = kx_i^2/2 - kx_f^2/2$$

From the equation, it is clear that the work done by the force of spring depends only on the endpoints of displacement. Also, we can see that in a cyclic process, the work done by the springs force is zero. Hence, we can say that the spring force is a conservative force because it depends on the initial and final positions only. Therefore, this work done is in the form of the Spring potential energy.

Solved Examples For You

Q: If a spring extends by x on loading, then the energy stored by the spring is (Given T is the spring force and K is force constant):

A) $2x/T^2$ B) $T^2/2k$ C) $2k/T^2$ D) $T^2/2x$

Solution: B) Force or tension, $T = kx$ [Hooke's Law]. Hence, $x = T/k$

Energy stored in the spring = $k\,x^2/2 = T^2/2k$

Power

We hear the word "power" quite frequently in day to day life. Cricket commentators talk about the Power behind a batsman's shot while broadcasting a cricket match, and football commentators may talk about a particularly powerful free kick by a player. We among ourselves have, at one point or another, arm-wrestled with a friend to decide who is more powerful!

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Power

Simply put is the rate at which work is done. Power, like work, is a scalar quantity. It is denoted by P and its SI unit is watt (W). This unit is named after Scottish scientist James Watt, who is credited with the invention of the steam engine. Thus, the average power formula over a time period is:

$$P_{av} = \frac{W}{t}$$

For example, a wrecking ball of mass 500 kg is dropped from a height of 50 metres on the ground. Let us calculate the power of its impact on the ground. From the law of conservation of mechanical energy, we know that $\Delta(K + V) = 0$. Let us try and solve this.

To find the Time

Here, $\Delta V = m\ g\ h = 500 * 9.8 * 50$ or, $\Delta V = \Delta K = 2{,}45{,}000J$

Since, it is a free fall, $W = \Delta K = 2{,}45{,}000J$. Therefore $K = (mv^2)/2$

Hence, $2{,}45{,}000 = (500 * v^2)/2$ or, $(2{,}45{,}000 * 2)/500 = v^2$

Which gives $v = \sqrt{98o} = 31.3$ m/s.

Since, for a free fall, $v = gt$, we have: $31.3 = 9.8 \times t$

or $t = 31.3/9.8 = 3.19s$

To find the Power

By applying the power formula, $P = W/t$

we have, here, $P = 2{,}45{,}000/3.19 = 76{,}802.5$ W

Hence, P = 76.8 KW. Hence, the power of the wrecking ball's impact on the surface is 76.8 kilowatt.

Power in Terms of Force and Velocity

We already know that power is the time rate of Work done. So, at any given point in time, the power can be defined as:

$$P = \frac{dW}{dt}$$

dW is the work done in the time period of dt. Since, we know that W = F.d, we can rewrite the above equation as:

$$P = F.\frac{dx}{dt}$$

Here, F is the force, dx is the displacement of the object and dt is the time period of that displacement. Since, we know that v = dx/t, we can rewrite the above equation as:

$$P = F.v$$

F is the force and v is the instantaneous velocity of the object. Force and velocity are the vector quantities and power is the scalar product of these two vectors. Hence, we have derived the power formula in terms of force and velocity. So,

$$P = \vec{F} . \vec{v}$$

$$P = F\ v\ cos\theta$$

Let us consider another example. While testing a newly developed engine, the engineers of Honda see that their 125 kg testing motorcycle propels from standstill to a top speed of 100 km per hour in 5 seconds. If the same engine is used in their racing motorcycle of mass 85 kg, how many seconds will elapse before the racing motorcycle achieves a speed of 100 km per hour?

Solution: We have: v = 100 × (1000/3600) = 27.78 m/s. We know that, a = v/t or average acceleration of the test motorcycle a = 27.78/5 = 5.56 m/s^2. We also know that, F = ma. Hence Force applied by engine is F = 125 × 5.56 = 695 N. By applying the power formula, P = F v, the peak power of the engine is P = 695 × 27.78 = 19,307 W

For the Racing Motorcycle

Since, the engine is the same, its peak power will remain constant. Therefore $P = 19{,}307W$. At peak power, the speed of racing motorcycle is same, or $v = 27.78$ m/s. By applying the power formula, $P = F v$ or, $19307 = F \times 27.78$, we get, $F = 19307/27.78 = 695$ N. Since, $F = m a$, we have $695 = 85 \times a$.

Therefore the acceleration of the racing motorcycle is $a = 695/85 = 8.18$ m/s^2. Hence the time required for the racing motorcycle to reach 100 kmph is, $t = v/a = 27.78/8.18$ or 3.4 seconds.

Power, Horsepower and Kilo-Watt-Hour

Other than the Watt, we encounter "horsepower" or hp as the most common unit to describe power. All cars and motorcycles use hp to describe the power of their engines. Also, most electric motors also use h.p. to describe their power rating.

$$1 \text{ hp} = 746 \text{ W}$$

Kilo-watt-hour or KWH is the unit of energy. It is used most commonly in our electric bills to denote the number of electrical units consumed. One kWh is the application of 1000 W of power for 1 hour. So,

$$1 \text{ KWH} = 1000 \text{ W} \times 3600 \text{ seconds} = 3600000 \text{ J}$$

Solved Examples for You

Q: A builder calls an engineer to install an elevator in his building. When the engineer inquires about the requirements, builder says that he wants the elevator to be able to carry a maximum of eight people at an average speed of 3 m/s. If the average load of a person is assumed to be 70 kg, and the elevator itself weighs 150 kg, find out the minimum power rating of the required motor in hp, if there is no friction involved.

If the total height of the top floor of the building is 50 m, find out the cost of transporting eight people to the top floor, if the cost of electricity is ₹11.65 per unit.

Solution: Mass of elevator with eight people = $8 \times 70 + 150 = 710$ kg. Hence, $F = 710 \times 9.8 = 6{,}958N$. Thus the minimum force required by the motor to overcome the gravity is $F = 6{,}958N$. By applying the power formula, $P = F\ v$, the minimum power rating of the required motor is $P = 6958 \times 2 = 13{,}916$ W.

Also, 1 hp = 746 W, therefore required hp rating of motor is = 13916/746 or, 18.65 hp. The height of the top floor is 50 m and the average speed of the elevator is 2 m/s. Also $v = d/t$ or, $t = 50/2$

The time taken by elevator to the top floor is 25 seconds. Electrical consumption in kWh = motor rating in kW × time in hours. Electrical units consumed in one trip = $13.916 \times (25/3600)$. Energy Units consumed in one trip = 0.097 kWh. Since the cost of one unit is ₹11.65 therefore the total cost of one trip in terms of electricity is, 11.65×0.097 or, ₹1.13

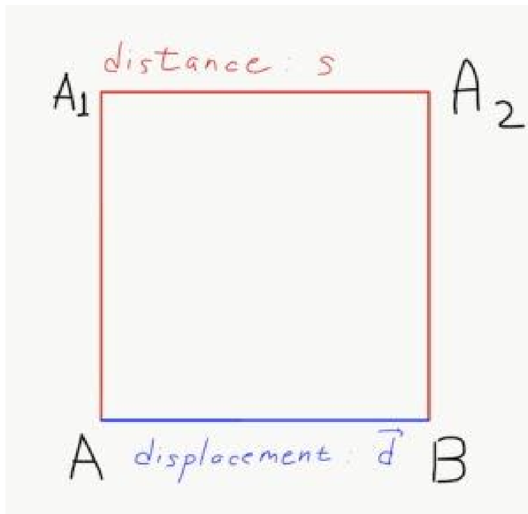
The Scalar Product

Velocity, displacement, force and acceleration are different types of vectors. Is there a way to find the effect of one vector on another? Can vectors operate in different directions and still affect each other? The answer lies in the Scalar product. An operation that reduces two or more vectors into a Scalar quantity! Let us see more!

What is a Scalar Quantity?

Before we learn about the Scalar product of two vectors, let's refresh what we have already learned about the difference between a vector quantity and a scalar quantity. Scalar quantity is one dimensional and is described by its magnitude alone. For example, distance, speed, mass etc.

Vector quantities, on the other hand, have a magnitude as well as a direction. For example displacement, velocity, acceleration, force etc. Some vector quantities are effectively the directional values of their corresponding scalar quantities. For example, displacement, effectively, is the distance in a particular direction. Vector quantities are expressed with an arrow above the appointed symbols. For example, displacement is expressed as $\vec{d}$. The following image explains the difference between distance, a scalar quantity, and displacement, a vector quantity:

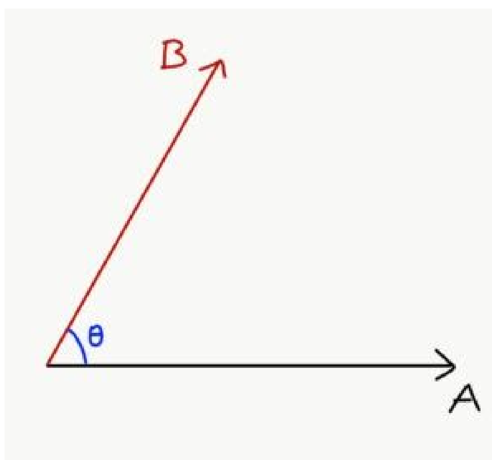


In the above diagram, if we take the square AA1A2B and assume it's side as 5m, then the object travelling the red path has travelled a distance of 15 meters. But, the displacement of the said object is only 5 meters in the direction of A to B. Similarly, if we assume that the same object has finished the traversing the red path in one second, then, the object's speed is 15 m/s. But, the velocity of the said object is only 5 m/s.

Now, in Physics, from time to time, we need to multiply two vector quantities. Some of these multiplications require a scalar product. For example, Work is a scalar quantity and is a product of Force and Displacement. Here, we will learn how to derive a scalar quantity as a product of two vectors, and, how these multiplications hold various laws of mathematics.

Scalar Product of Two Vectors

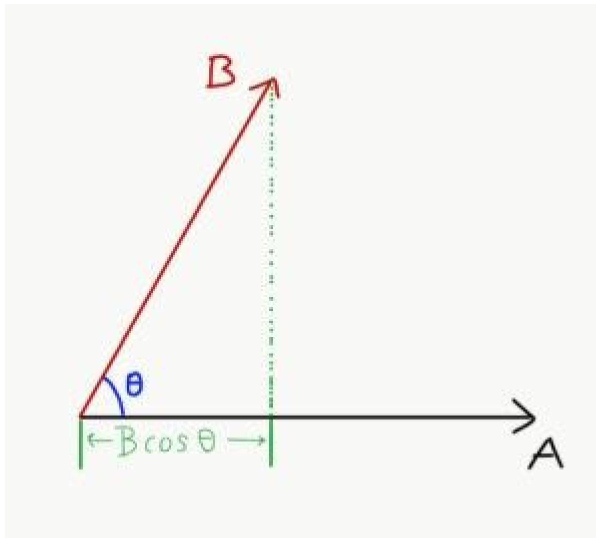
Let's consider two vector quantities A and B. We denote them as follows:

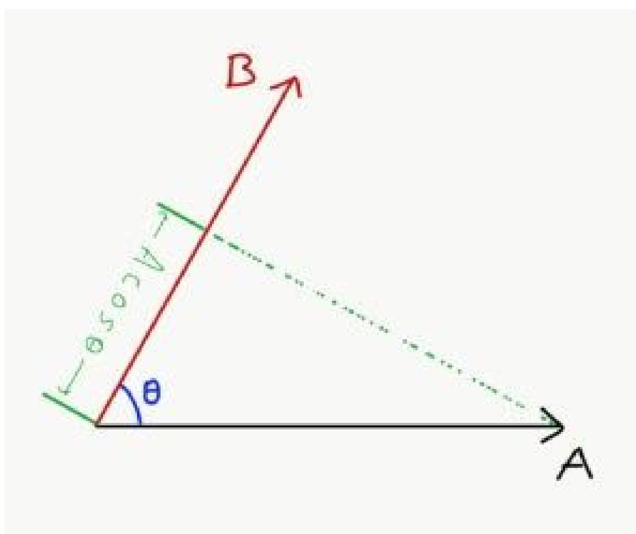


Their scalar product is A dot B. It is defined as:

$$A.B = |A|\ |B|\ \cos\theta$$

Where, θ is the smaller angle between the vector A and vector B. An important reason to define it this way is that |B|cosθ is the projection of the vector B on the vector A. The projections can be understood from the following images:





Since, A(Bcosθ) = B(Acosθ), we can say that

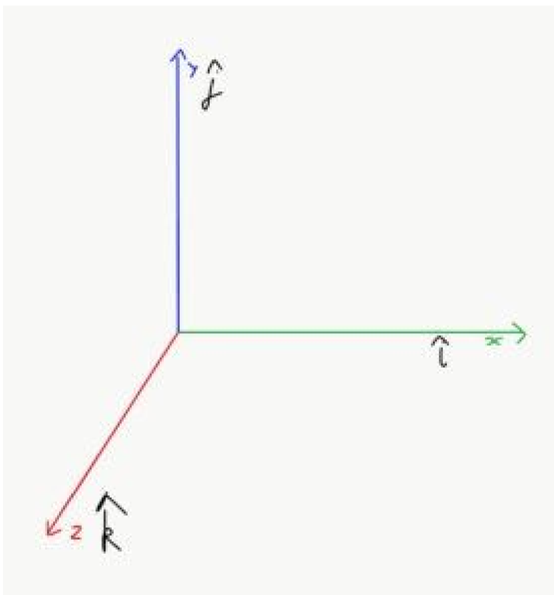
$$A.B = B.A$$

Hence, we say that the scalar product follows the commutative law.

Similarly, the scalar product also follows the distributive law:

$$A.(B+C) = A.B + A.C$$

Now, let us assume three unit vectors, i, j and k, along with the three mutually perpendicular axes X, Y and Z respectively.



As cos (0) = 1, we have:

$$\hat{i} \cdot \hat{i} = \hat{j} . \hat{j} = \hat{k} \cdot \hat{k} = 1$$

Also since the cosine of 90 degrees is zero, we have:

$$\hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{k}=\hat{k}\cdot\hat{i}=0$$

These two findings will help us deduce the scalar product of two vectors in three dimensions. Now, let's assume two vectors alongside the above three axes:

$$A=Ax\,\hat{i}+Ay\,\hat{j}+Az\,\hat{k}$$

$$B=Bx\,\hat{i}+By\,\hat{j}+Bz\,\hat{k}$$

So their scalar product will be,

$$A\cdot B=\left(Ax\,\hat{i}+Ay\,\hat{j}+Az\,\hat{k}\right).\left(Bx\,\hat{i}+By\,\hat{j}+Bz\,\hat{k}\right)$$

Hence,

$$A.B = A_xB_x + A_yB_y + A_zB_z$$

Similarly, A_2 or A.A = $$A^2 = A_x^2 + A_y^2 + A_z^2$$

In Physics many quantities like work are represented by the scalar product of two vectors. The scalar product or the dot product is a mathematical operation that combines two vectors and results in a scalar. The magnitude of the scalar depends upon the magnitudes of the combining vectors and the inclination between them.

Solved Examples For You

Q: Let's find the angle between force and the displacement; where, F = (2i + 3j + 4k) and d = (4i + 2j + 3k).

Solution: We already know that, $A.B = A_xB_x + A_yB_y + A_zB_z$

Hence, $F.d = F_xd_x + F_yd_y + F_zd_z = 2*4 + 3*2 + 4*3 = 26$ units

Also, F.d = F dcosθ

Now, $F^2 = 2^2 + 3^2 + 4^2 = \sqrt{29}$ units

Similarly, $d^2 = 4^2 + 2^2 + 3^2 = \sqrt{29}$ units

Hence, F d $\cos\theta = 26$

Then, $\cos\theta = 26/(F\ d) = 26/(\sqrt{29} \times \sqrt{29}) = 26/29$, which gives $\cos\theta = 0.89$ or $\theta = \cos^{-1}(0.89)$

Work and Kinetic Energy

We all have heard the stories of great explorers who sailed the unknown seas in their sailboats. Those were the days before any engine was in existence. They only relied on wind to move their huge ships. How does the wind move such objects? The answer lies in the principles of Kinetic energy. In this article, we shall understand its basic principles and how can it equate to work.

Kinetic Energy Definition

Kinetic Energy of an object is its capacity to put another object in motion. A force displacing an object does work. Also, we know that when an object in motion strikes a stationary object, it can cause it to move. Hence it can do work. Thus, we define the *kinetic energy as the capacity of an object to do work by virtue of its motion*.

The kinetic energy of an object is denoted by KE and its SI unit is joules (J). By now, we have started to sense that work and kinetic energy are closely related. But, are they interchangeable? Can work be expressed in terms of the kinetic energy of an object and vice versa? Let us see!

Browse more Topics under Work Energy And Power

- Collisions
- Concepts of Potential Energy
- Conservation of Mechanical Energy
- Potential Energy of a Spring
- Power
- The Scalar Product
- Work-Energy Theorem

- Various Forms of Energy: The Law of Conservation of Energy

If you want to learn more about Kinetic Energy click here.

Work and Kinetic Energy – The Work-Energy Theorem

Consider an object with an initial velocity 'u'. A force F, applied on it displaces it through 's', and accelerates it, changing its velocity to 'v'. Its equation of motion can be written as: $v^2 - u^2 = 2as$

Multiplying this equation by 'm' and dividing throughout by 2, we get:

$mv^2/2 - mu^2/2 = mas$; where 'm' is the mass of the object.

Hence, $mv^2/2 - mu^2/2 = Fs$; where F is the force that caused the havoc!

Therefore, we can write $mv^2/2 - mu^2/2 = W$; where $W = Fs$ is the work done by this force.

So what just happened? We just proved that $1/2\ (mv^2) - 1/2\ (mu^2)$ is the work done by the force! In other words, the work done is equal to the change in K.E. of the object! This is the Work-Energy theorem or the relation between Kinetic energy and Work done. In other words, the work done on an object is the change in its kinetic energy. $W = \Delta(K.E.)$

The engine of your motorcycle works under this principle. The explosion of the burning mixture of fuel and air moves the piston. The moving piston, in turn, moves the crank of the engine, which, in turn, moves the selected gear and hence the drive chain that rotates the driving wheel of the motorcycle.

Therefore, engineers use the work-energy theorem to calculate the work done by each progressive component of the engine in its chain of motion. By calculating the difference in work done, engineers can isolate the performance of each component of the drivetrain and attempt to improve its efficiency.

Solved Examples For You

Q 1: Which of the following kind of energy depends upon the mass and the square of the speed of a body?

A) Kinetic B) Potential C) Electrostatic D) Nuclear

Solution: A) The Gravitational Potential energy, Nuclear and Kinetic Energy all depend on mass. But out of the three only Kinetic energy is such that it contains the term with the square of velocity. So the answer is A.

Q 2: A rigid body of mass m kg is lifted uniformly by a man to a height of one metre in 30 sec. Another man lifts the same mass to the same height in 60 sec. The work done on the body against gravitation in both the cases are in the ratio:

A) 1:2 B) 1:1 C) 2:1 D) 4:1

Solution: B) Well whoever said anything about work depending on time, right? Work is independent of time as W = F.s doesn't contain time (as long as the force is time independent!). So we can say that in this case, the work done will be similar in both cases and will present a ratio of 1:1.

Work-Energy Theorem

If you transfer a certain amount of energy to an object in motion, what will happen to it? Can you measure the amount of energy in terms of work? How much work does it take to launch a satellite into space? Let us find out the Work-Energy Theorem and answer these questions!

Work-Energy Theorem

The term "work" is used in everyday life quite frequently and we understand that it's an act of doing something. For example, you are working right now on your grasp of Physics by reading this article! But, Physics itself might not agree on this. The Work-energy theorem explains the reasons behind this Physics of no work!

Work is said to be done when an acting force displaces a particle. If there is no displacement, there is no work done. You might get tired if you keep standing for a long time, but according to Physics, you have done zero work.

Thus, work is a result of force and the resulting displacement. Now, we already know that all moving objects have kinetic energy. So, there must be a relation between Work and kinetic energy. This relation between the kinetic energy of an object and workdone is called "Work-Energy Theorem". It is expressed as:

$$W = \Delta K$$

Here, W is the work done in joules (J) and ΔK is the change in kinetic energy of the object. To learn how the Work-Energy Theorem is derived, we must first learn the nature of work as a scalar quantity and how two or more vector quantities are multiplied.

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Proof of Work-Energy Theorem

We will look at the Work-Energy Theorem in two scenarios:

Workdone Under a Constant Force

We have already learnt about the equations of motion earlier and know that,

$v^2 = u^2 + 2as$ Here, v is the final velocity of the object; u is the initial velocity of the object; a is the constant acceleration and s is the distance traversed by the object. We can also write this equation as,

$$v^2 - u^2 = 2as$$

We can substitute the values in the equation with the vector quantities,

therefore: $v^2 - u^2 = 2a.d$

If we multiply both sides with m/2, we get:

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = ma.d$$

From Newton’s second law, we know that F=ma, hence:

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = F \cdot d$$

Now, we already know that W= F.d and, K.E. = $(mv^2)/2$,

So, the above equation may be rewritten as:

$$K_f - K_i = W$$

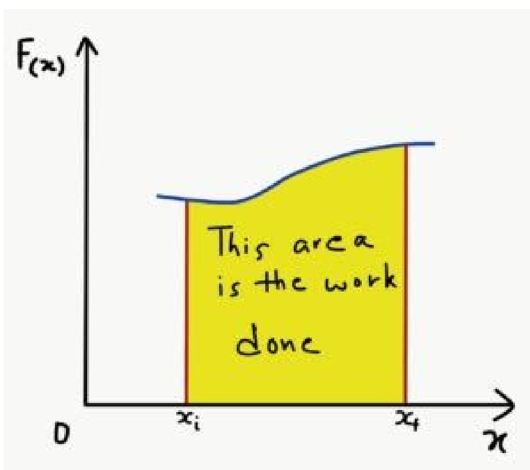
Hence, we have:

$$\Delta K = W$$

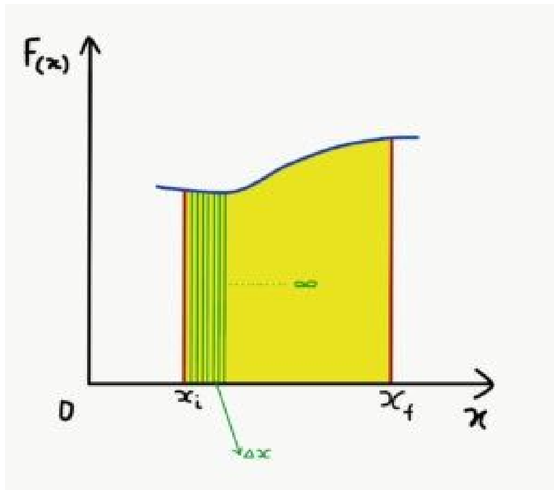
Therefore, we have proved the Work-Energy Theorem. The Work done on an object is equal to the change in its kinetic energy.

Workdone Under a Variable Force

A constant force is rare in the everyday world. A variable force is what we encounter in our daily life. Proving the Work-Energy Theorem for a variable force is a little tricky. If we take one axis as the Force applied and the other axis as displacement, we come up with the following graph:



Here, x is the displacement. Now let's divide this area into rectangles of infinitely small width along the x-axis:



Here, for an infinitely small displacement of "delta x", we can assume the force to be constant. Hence,

$$\Delta W = F(x)\Delta x$$

If we add all the rectangles in the second graph, we come up with the total work. It can be expressed as,

$$W = \sum_{x_i}^{X_f} F(x)\Delta x$$

If total number of rectangles approaches infinity, the width of these rectangles along the axis x will approach zero,

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{X_f} F(x) \Delta x$$

Part one: Now we can define the work done as a definite integral of the force over the total displacement as $W = \int_{x_i}^{xf} F(x) dx$ Now, we already know that kinetic energy of an object can be expressed as:

$K = \frac{1}{2} m v^2$ We can express the change in the kinetic energy with time as,

$$\frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2} m v^2\right)$$

Or,

$$\frac{dK}{dt} = m \frac{dv}{dt} v$$

We know from Newton's second law that acceleration is the change in velocity of an object with the rate of time. So the above equation can be alternatively written as,

$$\frac{dK}{dt}=mav$$

Since F=ma and the velocity of an object is the change in displacement over a time rate, the above equation can be alternatively written as: $\frac{dK}{dt}=F\frac{dx}{dt}$

By cancelling out the time derivative from both sides of the equation, we get,

$$dK=Fdx$$

If we integrate the above equation along the x-axis of the second graph, we get,

$$\int_{K_i}^{K_f} dK = \int_{x_i}^{xf} F dx$$

Or $K_f - K_i = \int_{x_i}^{xf} F dx$ Hence: $\Delta K = \int_{x_i}^{xf} F dx$ The right-hand side of the above equation denotes the work. Hence we get:

$$\Delta K = W$$

Hence, the work-energy theorem is proved for the variable force as well.

Solved Examples For You

Q 1: A bullet of mass 20 g moving with a velocity of 500 m/s, strikes a tree and goes out from the other side with a velocity of 400 m/s. Calculate the work done by the bullet (in joules) in passing through the tree:

A) 900 J B) 800 J C) 950 J D) 500 J

Solution: A) Given mass of the bullet, m = 20 g = 0.02 kg. Initial velocity of the bullet = 500 m/s. Final velocity of the bullet = 400 m/s. Now if we can calculate the change in energy of the bullet or in other words the work done by the bullet on the tree, we have:

Hence, using the Work-Energy Theorem, we have: Δ(K.E.) of the bullet = $1/2\{0.02(500)^2 – 0.02(400)^2\}$

Therefore, Δ(K.E.) of the bullet = 900 J

Various forms of Energy: The Law of Conservation of Energy

Energy is simply the capacity of an object to do work. Energy change occurs during most events and processes in the Universe. In all of these Energy, conservation is followed! Our Universe has a fixed amount of energy. Energy can neither be created nor be destroyed, it may change form. Here, we will learn about various forms of energy with a special focus on the energy conservation.

Mechanical Energy

The capacity of an object to do work by virtue of its motion or position is its Mechanical Energy. It is the sum of the kinetic energy and potential energy. Since in most of the physical processes, we usually come across Kinetic and Potential energy only, we can use it to study energy conservation.

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- Work-Energy Theorem

Kinetic Energy

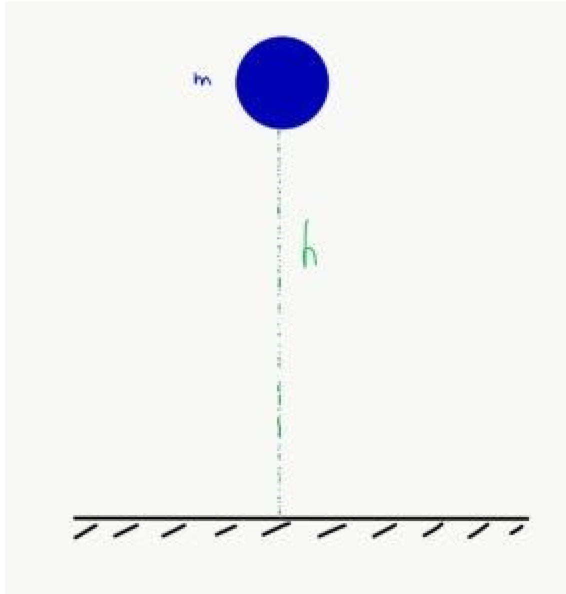
The ability of an object to do work because of its motion is called Kinetic Energy. For example, the wind's kinetic energy pushes the sail of a boat and the boat moves forward. Flowing water has kinetic energy, and it has been harnessed to drive the grinders of flour mills for centuries. Kinetic Energy is denoted by K and its unit is joules (J). It is expressed as:

$K = \frac{1}{2}mv^2$ Here, K is the kinetic energy of the object, m is the mass of the object and v is the velocity of the object

Potential Energy

Potential energy can be defined as the capacity of an object to do work by virtue of its position. For example, a stretched bow string possesses potential energy. When the bowstring is released, it propels the arrow forward. Another example might be that of an object raised above the ground. When the said object is released, it rushes downwards.

A compressed spring also possesses potential energy, when it is released, it expands with a force. Potential Energy is denoted by V and its unit is joules (J). Here, we will concentrate on the potential energy of an object by virtue of its position with respect to the earth's gravity. Let us consider the following illustration:



Here, m is the mass of the object in kilograms and h is the height of the object from earth's surface in meters. Here, the potential energy of the object at a height of h can be expressed as:

$$V(h) = mgh$$

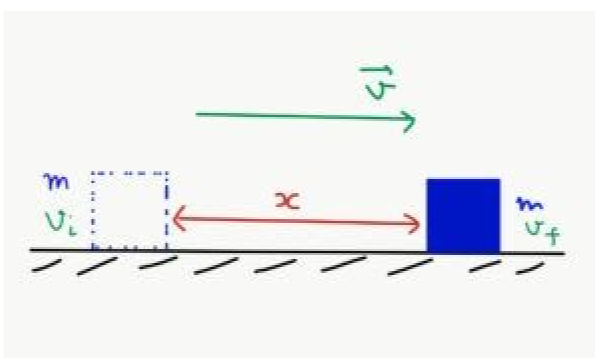
Here, g is considered to be the earth's gravitational constant and its value is set at $9.8 m/s^2$. We know that an object will accelerate at different rates with respect to its distance from earth's center of gravity. But the surface heights are minuscule as compared to the earth's radius, and hence, for all practical purposes, the acceleration under the earth's gravitational force is taken to be a constant.

In the physical world, energy is transferred in many forms. Let us discuss some of these forms.

Heat Energy

Heat energy is associated with the frictional force. For example, if we rub our hands together in the winter, they feel warm. Similarly, the tip of a dentist's drill gets extremely heated while drilling into a tooth. So a cooling mechanism is incorporated in the drill that regulates the temperature by a jet of water.

Let us consider the following illustration where we consider an object in motion. Let us say that this object comes to a complete rest by virtue of the friction of the surface alone.



Here m is the mass of the object, v_i is the initial velocity of the object, x is the displacement of the object and v_f is the final velocity of the object $v_f = 0$. After we apply the work-energy theorem, work done by the frictional force of the surface will be:

$W_{(friction)} = \Delta K$

$W_{(friction)} = K_f - K_i$; since, $K_f = 0$ then, $W_{(friction)} = - K_i = 1/2\ [mv_i^2]$

We also notice that work done by friction, in this case, is negative. The frictional force transforms the kinetic energy of the block into heat energy. This heat energy increases the temperature of both the object as well as the surface.

Chemical Energy

Chemical energy can be most simply defined as the energy that binds together the atoms and molecules of various materials. When these molecular bonds are broken, a large amount of energy is released. For example, when we light a fire to some wooden logs, we break down the complex organic molecules, and their chemical energy is released. Basically, a chemical reaction is carried by the transaction of energy. On which basis there are two types of reactions:

- Exothermic reactions: A chemical reaction that releases energy. For example, oxidation of coal. A kilogram of coal releases 3×10^7 J of energy when burnt.
- Endothermic reaction: A chemical reaction that absorbs energy. For example, the reaction of ammonium nitrate and water requires heat to proceed.

Nuclear Energy

Nuclear energy binds together the protons and neutrons in the nucleus of each element. It is the strongest force in the universe. Nuclear bombs harness the nuclear energy of uranium and plutonium. The devastation of Hiroshima and Nagasaki at the end of the Second World War was caused by a minuscule amount of nuclear matter when compared to the traditional bombing materials. Nuclear reactions can be categorized into:

- Fission: When a larger nucleus disintegrates into smaller nuclei. For example, the fission of uranium nucleus into smaller nuclei like thorium and radium etc.

- Fusion: When two or more smaller nuclei fuse together to form a bigger nucleus. For example, our sun's energy is derived by the fusion of hydrogen atoms.

Electrical Energy

Charges exert forces on each other and hence give rise to an electrical energy. The flow of electrical current has energy. This energy can be harnessed by passing the electrical current through various materials and apparitions. For example, when electrical current passes through the filament of a bulb, it produces light. And when electrical current is passed through the motor of a fan, it rotates the blades.

Law of Conservation of Energy

The law of energy conservation states that energy we can neither create nor destroy energy. Energy merely changes its form. For example, the kinetic energy of a moving object transforms into heat energy when the object comes to a full rest by the virtue of friction.

Broader perspective

Einstein proposed that matter and energy are interchangeable and their relation can be expressed as $E = mc^2$. Here, E is the total energy of the matter, m is the mass of the matter and c is the velocity of light in vacuum. C is a constant at 300000000 m/s. Thus, a mere kilogram of matter has the equivalent energy of 9×10^{16} J or the equivalent annual output of a large power station (3000 MW capacity)!

Energy Conservation Under the Confinement of Mechanical Energy

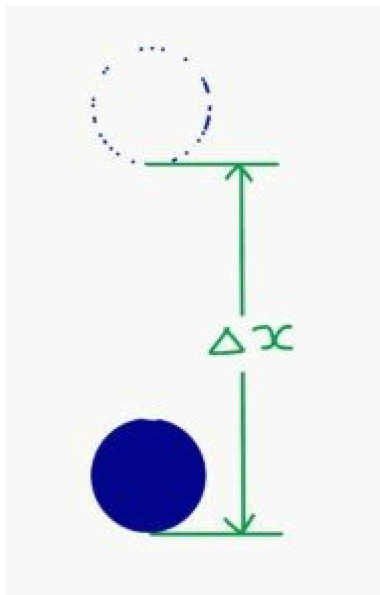
Let us confine ourselves to the boundaries of mechanical energy under a conservative force. Here, we can prove the law of energy conservation. Before we do that, let us understand the nature of a conservative force. A conservative force has following characteristics:

- We get it from a scalar quantity. Hence, it is one dimensional.
- Work done by a conservative force depends only on end points of the motion.
- If the endpoints of the motion are same i.e. the motion is in a closed loop, the work done by a conservative force is zero.

Now, let's consider the following illustration:

Solved Example for You

Example: Suppose an object falls from a height Δx towards the ground. Evaluate the energy conservation in this system.



Solution. Here, Δx is the displacement of the object under the conservative force F. By applying the work-energy theorem, we have

$\Delta K = F(x)\ \Delta x$

Since the force is conservative, the loss of potential Energy can be defined as $\Delta V = F(x)\ \Delta x$

It means: $\Delta K + \Delta V = 0$

Therefore: $\Delta(K + V) = 0$

This means, that for every displacement of Δx, the difference between the sums of an object's kinetic and potential energy is zero. In other words, the sum of an object's kinetic and potential energies is constant under a conservative force.