

# Ampere's Circuital Law

What is Ampere's Circuital Law? Well, it is a current distribution which helps us to calculate the magnetic field. And yes, the Biot-Savart law does the same but Ampere's law uses the case high symmetry. We will first understand the ampere's circuital law, followed by its proof. So let us begin!

## Ampere's Circuital Law

What is stated by Ampere's Circuital Law? The formula for this is a closed loop integral. The integral of magnetic field density (B) along an imaginary closed path is equal to the product of current enclosed by the path and permeability of the medium. Line integral to the magnetic field of the coil =  $\mu_0$  times the current passing through it. It is mathematically expressed as

$$\oint B \cdot dl = \mu_0 I$$

Here  $\mu_0$  = permeability of free space =  $4 \pi \times 10^{-7} \text{ N/A}^2$  and  $\oint B \cdot dl$  = line integral of B around a closed path.

## Proof of Ampere's Circuital Law

### Case 1: Regular Coil

Consider a regular coil, carrying some current  $I$ . Let us assume a small element  $dl$  on the loop.

$$\int B \, dl = \int B \, dl \cos \theta$$

Here,  $\theta$  is the small angle with the magnetic field. The magnetic field will be around the conductor so we can assume,

$$\theta = 0^\circ$$

We know that, due to a long current-carrying wire, the magnitude of the magnetic field at point  $P$  at a perpendicular distance ' $r$ ' from the conductor is given by,

$$B =$$

$$\mu$$

$$0$$

$$i$$

$$2\pi r$$

The magnetic field doesn't vary at a distance  $r$  due to symmetry. The integral of an element will form the whole circle of the circumference  $(2\pi r)$ :

$$\int dl = 2\pi r$$

Put the value of  $B$  and  $\int dl$  in the equation, we get:

$$B \int dl =$$

$$\mu$$

$$0$$

$$i$$

$$2\pi r$$

$$\times 2\pi r = \mu oi$$

$$\text{therefore, } \int B \cdot dl = \mu oi$$

Case 2: Irregular Coil

Irregular coil means a coil of any arbitrary shape. Here the radius will not remain constant as it is not a regular coil.

$$\int \mathbf{B} \cdot d\mathbf{l}_1 = \int$$

$$\mu$$

$$0$$

$$i$$

$$2\pi r$$

$$\times d\mathbf{l}_1$$

As we know :  $d\theta_1 =$

$$d$$

$$l$$

$$1$$

$$r$$

$$1$$

$$\therefore \int$$

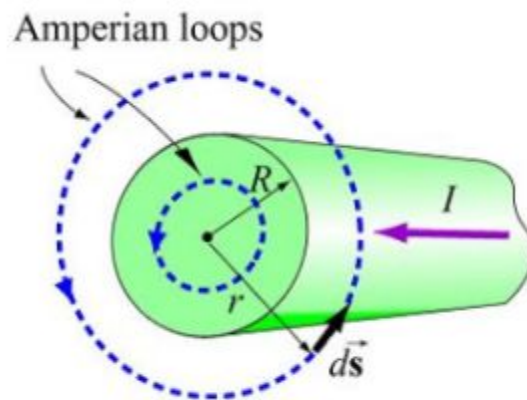
$\mu$ 
 $0$ 
 $i$ 
 $2\pi r$ 
 $\times dl_1 =$ 
 $\mu$ 
 $0$ 
 $i$ 
 $2\pi$ 

$$\int d\theta_1 = \mu_0 i$$

$$\int \mathbf{B} \cdot d\mathbf{l} = \mu_0 i$$

So whether the coil is a regular coil or an irregular coil, the ampere's circuital law holds true for all.

## Amperian Loop



Ampere's circuit law uses the Amperian loop to find the magnetic field in a region. The Amperian loop is one such that at each point of the loop, either:

- $B$  is tangential to the loop and is a non zero constant
- or  $B$  is normal to the loop, or
- $B$  vanishes

where  $B$  is the induced magnetic field.

## Solved Examples for You

Q1. Mark the incorrect option.

- A. Amperes law states that the flux  $B$  through any closed surface is  $\mu_0$  times the current passing through the area bounded by a closed surface.
- B. Gauss's law of magnetic field serves the same purpose as the Gauss's law for the electric field.
- C. Gauss's law of magnetic field states that the flux of  $B$  in any closed surface is equal to zero, whether there are or not any currents within the surface.
- D. All of the above.

Solution: A. Ampere law states that for any close looped path, the sum of the length elements times the magnetic field in the direction of the length element is equal to the permeability times the electric current enclosed in the loop. Option A is correct.

Q2. A student gets confused if two parallel wires carrying current in the same direction attract or repel. Which rules will he need to reach the right conclusion?

- A. Right-Hand Thumb Rule
- B. Fleming Heft Hand Rule
- C. Both A and B

D. None

Solution: C. Consider two parallel wires carrying current in the same direction. When right-hand thumb rule and Fleming left-hand rule is applied, it is observed that the force in the direction of the first wire i.e second wire is attracted to the second wire. Similarly, the second wire is also attracted to the first wire. Hence they attract.

## Magnetic Field Due to a Current Element, Biot-Savart Law

We all know that **magnetic field** is produced by the motion of **electric charges** or **electric current**. Biot-Savart law gives this **relation** between current and magnetic field. It relates the magnetic field to the magnitude, direction, length, and proximity of the electric current.

### Biot-Savart Law

A small current carrying **conductor** of length  $dl$ , carrying a current  $I$  is an elementary source of magnetic field. The force on another similar conductor can be expressed conveniently in terms of magnetic field  $dB$  due to the first. The dependence of magnetic field  $dB$  on the



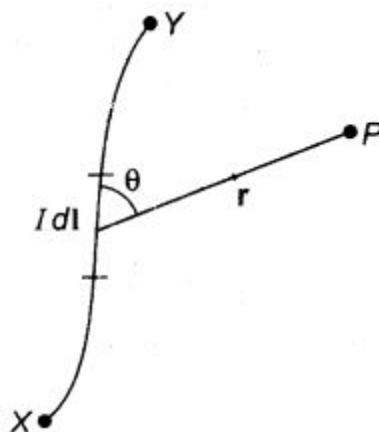
current  $I$ , on size and orientation of the length element  $dl$  and on distance  $r$  was first guessed by Biot and Savart.

## Mathematical Representation

The Biot-Savart's law gives the magnetic field produced due to a current carrying segment. This segment is taken as a vector quantity known as the current element.

Consider a wire carrying a current  $I$  in a specific direction as shown in the figure. Take a small element of the wire of length  $dl$ . The direction of this element is along that of the current so that it forms a vector  $I dl$ . If we want to know the magnetic field produced at a point due to this small element, then we can use the Biot-Savart's Law.

The magnitude of the magnetic field  $dB$  at a distance  $r$  from a current carrying element  $dl$  is found to be proportional to  $I$  and to the length  $dl$ . And is inversely proportional to the square of the distance  $|r|$ . The direction of the Magnetic Field is perpendicular to the line element  $dl$  as well as radius  $r$ .



(Source: learnCBSE)

Thus the vector notation is given as,  $d\vec{B} \propto Idl \times \vec{r} / r^3 = (\mu_0 / 4\pi) \times (Idl \times \vec{r} / r^3)$ , where  $\mu_0/4\pi$  is a constant of proportionality. The above [expression](#) holds when the medium is a vacuum. Therefore the magnitude of this field is:

$$|dB| = (\mu_0 / 4\pi) \times (Idl \sin\theta / r^2)$$

Learn more about [the Magnetic Force and Magnetic Field](#).

## Similarities And Differences Between Biot-Savart Law And Coulomb's Law

Similarities

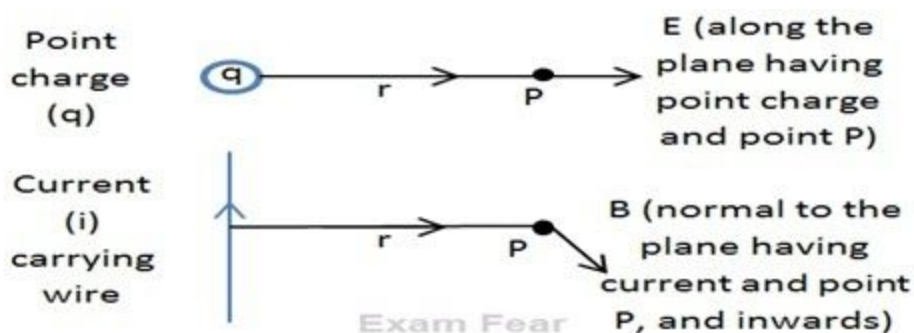
- Both magnetic and electric fields at a point are inversely proportional to the square of the distance between the field source and the point in question.
- Electric field due to a point charge (Coulomb's law) is:  $E = (1/4\pi\epsilon_0) \times (q/r^2)$
- Magnetic field due to a moving charge (Biot-Savart law) is:  $B = (\mu_0/4\pi) \times Idl (\sin\theta)/r^2$

Learn more about the [Motion in Combined Electric and Magnetic Field](#).

### Differences

- The source of the electrostatic field is [scalar](#) in nature. Whereas, the source of the magnetic field, which is the current element ( $Idl$ ), is a vector in nature.
- The electric field always acts along the plane containing distance ( $r$ ) between a point charge and the point where the electric field is to be calculated. But, the magnetic field acts in the plane perpendicular to the plane of distance( $r$ ) between the current element and the concerned point.

- Magnetic field depends on both the angle ( $\theta$ ) between the current element ( $Idl$ ) and the line joining the point and current element. However, the electric field doesn't depend on the angle ( $\theta$ ).



(Source: ExamFear)

The first diagram shows the electric field ( $E$ ) due to a point charge ( $q$ ). The second diagram shows the magnetic field ( $B$ ) due to the current carrying wire.

Learn more about [Domestic Electric Circuits](#).

## Solved Examples For You

Question: A circular coil is of 10 turns and radius 1m. If a current of 5A flows through it, calculate the field in the coil from a distance of 2m.

- A)  $314.16 \times 10^{-7} \text{ T}$     B)  $341.61 \times 10^{-7} \text{ T}$     C)  $200 \times 10^{-7} \text{ T}$     D)  
 $314.16 \times 10^{-10} \text{ T}$

Solution: Given: No. of turns  $n = 10$ , Current  $I = 5\text{A}$ , length  $l = 2\text{m}$ , radius  $r = 1\text{m}$

The Biot-Savart law formula is given by,

$$B = (\mu_0 / 4\pi) \times (2\pi nI / r)$$

$$\text{Therefore, } B = (\mu_0 / 4\pi) \times (2 \times \pi \times 10 \times 5 / 1)$$

$$B = 314.16 \times 10^{-7} \text{ T}$$

## Magnetic Force and Magnetic Field

As children, I am sure all of us have played with [magnets](#). Magnets have always been a mystery to us. They are a fun manipulative. At some orientation, they would pull each other towards themselves and

at some, they would move away from each other. Over the years we learnt that the force that works behind this behaviour of magnets is its **Magnetic Force** which is attractive or repulsive in nature depending on its orientation with other magnets.

## Magnetic Force

Magnetic Force can be defined as the attractive or repulsive force that is exerted between the poles of a magnet and electrically charged moving particles. It is a consequence of the electromagnetic force.

We have seen that the interaction between two charges can be considered in two stages. The **charge**  $Q$ , the **source** of the field, produces an **electric field**  $E$ , wherev

$E$

$\rightarrow$

$= Q$

$r$

$\rightarrow$

$$\frac{1}{(4\pi\epsilon_0)} \frac{qQ}{r^2},$$

$\hat{r}$

$\rightarrow$

$\hat{r}$  is unit vector along  $r$ , and the field  $E$  is a vector field. A charge  $q$  interacts with this field and experiences a force  $F$  given by

$F$

$\rightarrow$

$$= q$$

$E$

$\rightarrow$

$$= q Q$$

$r$

$\rightarrow$

$$/ ( 4 \pi \epsilon_0 ) r^2$$



(source: flickr)

## Magnetic Field

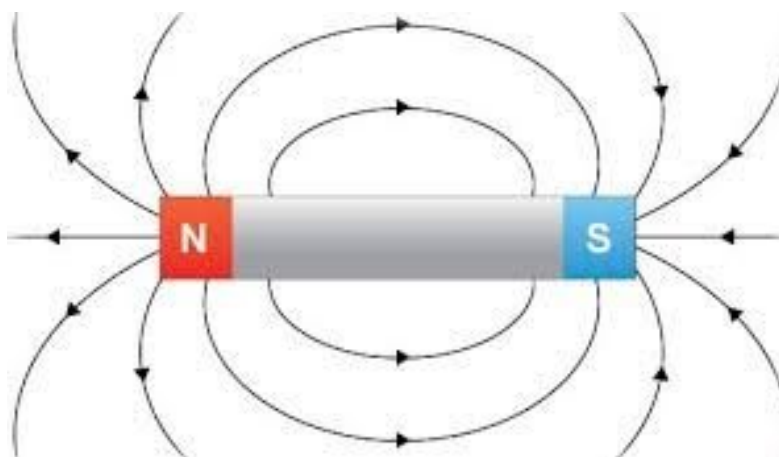
The Magnetic Field is the space around a magnet or current carrying conductor around which **magnetic effects** can be experienced. It is a vector **quantity** and its SI unit is Tesla (T) or  $\text{Wbm}^{-2}$

## Magnetic Lines of Force

It can be defined as curved lines used to represent a magnetic field, drawn such that the number of lines relates to the magnetic field's



strength at a given point and the tangent of any **curve** at a particular point is along the direction of magnetic force at that point.



*[source: qsstudy]*

## Properties

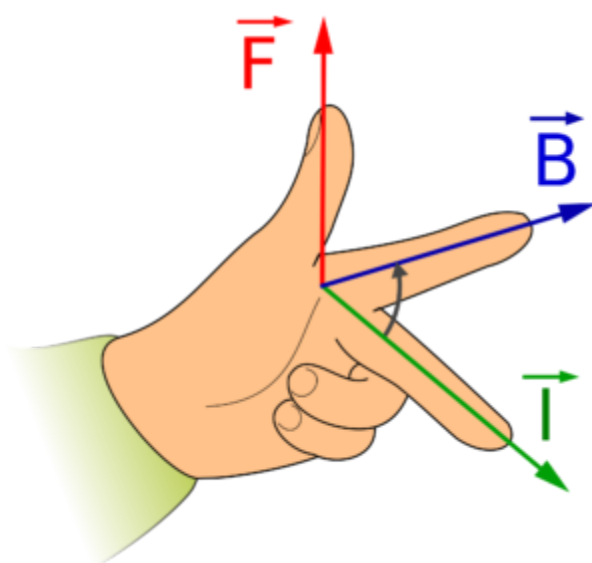
1. Magnetic lines of force start from the North Pole and end at the South Pole.
2. They are continuous through the body of a magnet.
3. Magnetic lines of force can pass through iron more easily than **air**.
4. Two magnetic lines of force can not intersect each other.
5. They tend to contract longitudinally.
6. They tend to expand laterally.

## Magnetic Force on Current-Carrying Conductor

A current-carrying conductor experiences a magnetic force in a magnetic field. Fleming's Left-Hand Rule predicts the direction of magnetic force,

$$F = IlB\sin\theta$$

where  $F$  is the magnetic force,  $I$  is current,  $l$  is the length of a straight conductor in a uniform magnetic field  $B$  and  $\theta$  is the angle between  $I$  and  $B$ .



Magnetic Force on Current-Carrying Conductor

## Solved Examples For You

Question: If a charged particle projected in a gravity-free room deflects, then

- A) There must be an electric field      B) There must be a magnetic field.
- C) Both fields cannot be zero      D) None of these

Solution: Since there must be some external force which will cause the deflection of charged particle and it can be both magnetic force or electric force. Therefore, simultaneously both the fields cannot be zero, therefore, option (C) is the answer. Also, option (A) and (B) are saying that there should be electric field compulsory or magnetic field compulsory for deflection which is not true, therefore, the only option is (C).

## Motion in Combined Electric and Magnetic Fields

It has long been known that charged **particles** move in **circular** orbits in the **magnetic field**. The Van Allen **radiation** belts in space around the earth consist of these energetic charges trapped in the magnetic

field of the earth. But what is a magnetic field? Magnetic fields are also used to guide the motion of charged particles in accelerators for both research and medical purposes. The orbiting **motion** of charges in a magnetic field is the basis for measuring the mass of an **atom**.

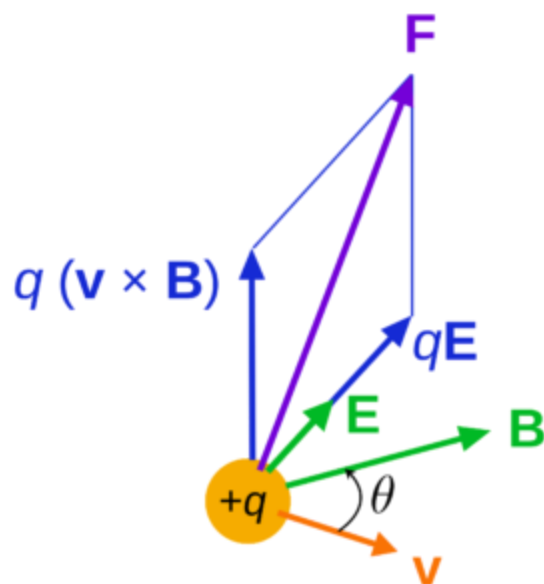
## Lorentz Force

Lorentz force is the force exerted on a charged particle moving through both electric and magnetic field.

$$F = qE + qv \times B \dots\dots\dots(1)$$

where,

- $F$  = Lorentz Force
- $q$  = Charge on the Particle
- $E$  = Electric Field
- $B$  = Magnetic Field
- $v$  = Velocity of the Particle



Lorentz Force

In a vacuum where collisions between particles are not very frequent, a particle with charge  $q$ , mass  $m$ , and velocity  $v$  perpendicular to a uniform magnetic field  $B$  (no  $E$ ) moves in a circular path with the radius

$$r = mv / qB \dots\dots\dots(2)$$

One can also deflect the trajectory of a charged particle with an electric field, although not into a circular path. If the electric force on the particle is both equal and opposite to the magnetic force, the net force on the particle will be zero. From Eq. (1), this will happen if

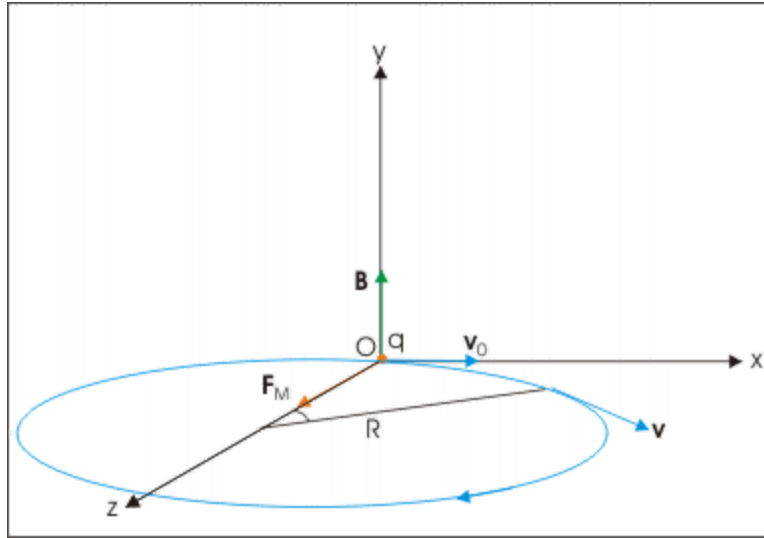
$$v = E / B \dots\dots\dots(3)$$

## The motion of a charged particle in the electric and magnetic field

In case of motion of a charge in a magnetic field, the magnetic force is perpendicular to the velocity of the particle. So no work is done and no change in the magnitude of the velocity is produced (though the direction of momentum may be changed). We shall consider the motion of a charged particle in a uniform magnetic field. First, consider the case of  $v$  perpendicular to  $B$ .

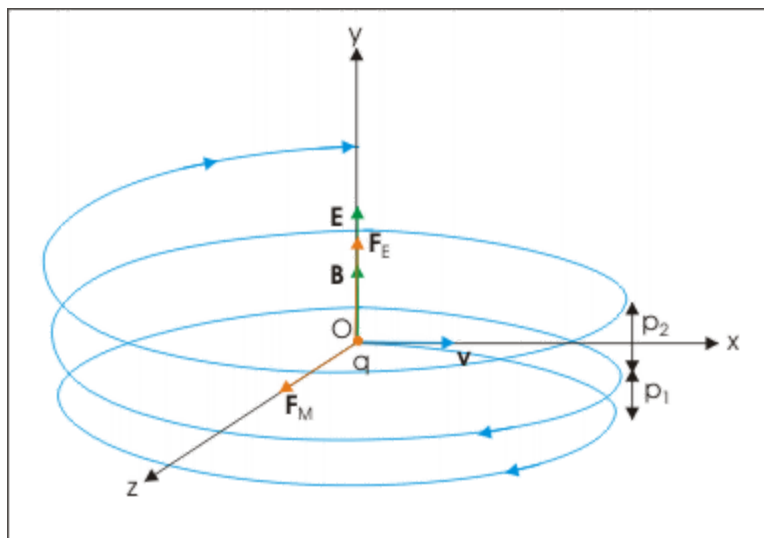
Learn more about [Magnetic Force and Magnetic Field](#).

The perpendicular force,  $q \mathbf{v} \times \mathbf{B}$ , acts as a centripetal force and produces a circular motion perpendicular to the magnetic field. If velocity has a component along  $B$ , this component remains unchanged as the motion along the magnetic field will not be affected by the magnetic field.



The circular motion of a charged particle in the magnetic field

The motion in a plane perpendicular to  $B$  is as before a circular one, thereby producing a helical motion. However, the electric field in  $y$ -direction imparts acceleration in that [direction](#). The particle, therefore, acquires velocity in the  $y$ -direction and resulting motion is a helical motion.



The motion of a charged particle in both electric and magnetic fields. Resulting motion is a helical motion with increasing pitch.

The radius of each of the circular element and other periodic attributes like time period, frequency and angular frequency is same as for the case of circular motion of a charged particle in perpendicular to magnetic field.

$$R = v / \alpha B$$

$$T = 2\pi / \alpha B$$

$$v = \alpha B / 2\pi$$

$$\omega = \alpha B$$

If there is a component of the velocity parallel to the magnetic field (denoted by  $v_2$ ), it will make the particle move along both the field and the path of the particle would be a helical one. The distance moved along the magnetic field in one rotation is called pitch  $p$ .

$$p = v_2 T = 2\pi m v_2 / qB$$

Learn more about [Domestic Electric Circuits](#).



## Applications

Some of the important applications associated with the presence of the two fields include :

- The motion of a charged particle in electric and magnetic fields
- Measurement of specific charge of an electron (J.J.Thomson experiment)
- Acceleration of charged particles (cyclotron)

Learn about [Torque on Current Loop, Magnetic Dipole](#).

## Solved Examples For You

Question: A charged particle moves in a gravity-free space without the change in velocity. Which of the following is/are possible?

- A)  $B = 0, E = 0$       B)  $E = 0, B \neq 0$       C)  $E \neq 0, B = 0$       D)  $B \neq 0, E \neq 0$

Solution: If A charged particle moves in a gravity-free space without a change in velocity, then

- Particle can move with constant velocity in any direction. So  $B = 0, E = 0$
- Particle can move in a circle with constant speed. Magnetic force will provide the centripetal force that causes particle to move in a circle.
- If  $qE = qvB$  and Magnetic & Electric force in opposite direction in this case also particle move with uniform speed.

## The Moving Coil Galvanometer

Have you ever wondered how the utility company knows how much power you use each month? In short, it uses an electric meter. The galvanometer is an instrument used to determine the presence, direction, and strength of an electric current in a conductor.

When an electric current is passing through the conductor, the magnetic needle tends to turn at right angles to the conductor so that its direction is parallel to the lines of induction around the conductor and its north pole points in the direction in which these lines of

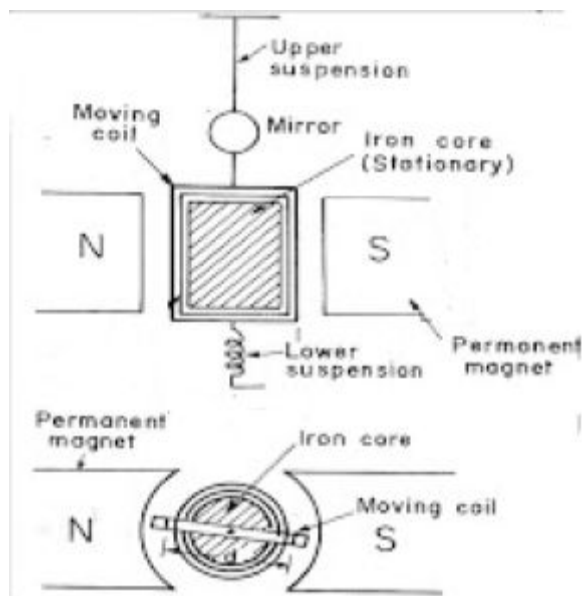
**induction** flow. A galvanometer is a type of ammeter. It is an instrument for detecting and measuring electric current.

## Moving Coil Galvanometer

Moving coil galvanometer is an **electromagnetic** device that can measure small values of current. It consists of **permanent horseshoe magnets**, coil, soft iron core, pivoted spring, non-metallic frame, scale, and pointer.

### Principle

**Torque** acts on a current carrying coil suspended in the uniform magnetic field. Due to this, the coil rotates. Hence, the deflection in the coil of a moving coil galvanometer is directly proportional to the current flowing in the coil.



[source: Redefining The Knowledge]

## The Moving Coil Galvanometer

### Construction

It consists of a rectangular coil of a large number of turns of thinly insulated copper wire wound over a light metallic frame. The coil is suspended between the pole pieces of a horseshoe magnet by a fine phosphor – bronze strip from a movable torsion head. The lower end of the coil is connected to a hairspring of phosphor bronze having only a few turns.

The other end of the spring is connected to a binding screw. A soft iron cylinder is placed symmetrically inside the coil. The

hemispherical magnetic poles produce a radial [magnetic field](#) in which the plane of the coil is parallel to the magnetic field in all its positions. A small plane mirror attached to the suspension wire is used along with a lamp and scale arrangement to measure the deflection of the coil.

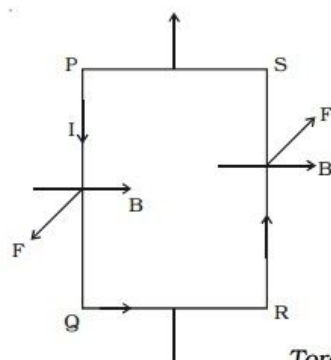
Learn about [Magnetic Field Due to Current Element, Biot-Savart Law](#)

### Working

Let PQRS be a single turn of the coil. A current  $I$  flows through the coil. In a radial magnetic field, the plane of the coil is always parallel to the magnetic field. Hence the sides QR and SP are always parallel to the field. So, they do not experience any force. The sides PQ and RS are always perpendicular to the field.

$PQ = RS = l$ , length of the coil and  $PS = QR = b$ , breadth of the coil.

Force on PQ,  $F = BI(PQ) = BI l$ . According to Fleming's left-hand rule, this force is normal to the plane of the coil and acts outwards.



Torque on the coil

Fig 3.28

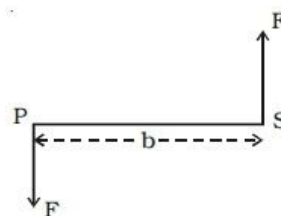


Fig 3.29

Force on RS,  $F = BI(RS) = BIl$ . This force is normal to the plane of the coil and acts inwards. These two equal, oppositely directed parallel forces having different lines of action constitute a couple and deflect the coil. If there are  $n$  turns in the coil, the moment of the deflecting couple =  $n BIl - b$

Hence the moment of the deflecting couple =  $nBIA$

When the coil deflects, the suspension wire is twisted. On account of elasticity, a restoring couple is set up in the wire. This couple is proportional to the twist. If  $\theta$  is the angular twist, then, the moment of the restoring couple =  $C\theta$ , where  $C$  is the restoring couple per unit twist. At [equilibrium](#), deflecting couple = restoring couple  $nBIA = C\theta$

Hence we can write,  $nBIA = C\theta$

$I = (C / nBA) \times \theta$  where  $C$  is the torsional constant of the spring; i.e. the restoring torque per unit twist. The deflection  $\theta$  is indicated on the scale by a pointer attached to the spring.

Learn about [Magnetic Force and Magnetic Field here](#)

## The sensitivity of Moving Coil Galvanometer

The sensitivity of a Moving Coil Galvanometer is defined as the ratio of the change in deflection of the galvanometer to the change in current. Therefore we write, Sensitivity =  $d\theta/di$ . If a galvanometer gives a larger deflection for a small current it is said to be sensitive.

The current in Moving Coil galvanometer is:  $I = (C/nBA) \times \theta$

Therefore,  $\theta = (nBA/C) \times I$ . Differentiating on both sides wrt  $I$ , we have:  $d\theta/di = (nBA/C)$ . The sensitivity of Moving Coil Galvanometer increases by:

- Increasing the no. of turns and the area of the coil,
- Increasing the magnetic induction and

- Decreasing the couple per unit twist of the suspension fibre.

## Advantages and Disadvantages

### Advantages

- Sensitivity increases as the value of  $n$ ,  $B$ ,  $A$  increases and value of  $k$  decreases.
- The eddy currents produced in the frame bring the coil to rest quickly, due to the coil wound over the metallic frame.

### Disadvantages

- Its sensitivity cannot be changed at will.
- Overloading can damage any type of galvanometer.

Learn more about Magnetism:

- [AC Generator: Parts, Working Mechanism, Phases, Videos, and Examples](#)
- [Domestic Electric Circuits](#)



- [Motion in Combined Electric and Magnetic Fields](#)

## Solved Examples for You

Question: Assertion: The resistance of a milliammeter is greater than that of the ammeter

Reason: Shunt resistance in case of a milliammeter is more than that of the ammeter.

- A. Both (A) and (R) are true and (R) is the correct explanation of (A).
- B. Both (A) and (R) are true but (R) is not the correct explanation of (A).
- C. (A) is true but (R) is false.
- D. (A) is false but (R) is true.

Solution: Unlike voltmeter, to have a more accurate reading in the ammeter, the whole current should pass through the ammeter for which the shunt resistance should be much high. Therefore the resistance of milliammeter is more than just ammeter.

Question: Moving Coil Galvanometer uses phosphor-bronze wire for suspension because it has

- A. High Conductivity
- B. High Sensitivity
- C. A large couple per unit twist
- D. Small couple pr unit twist

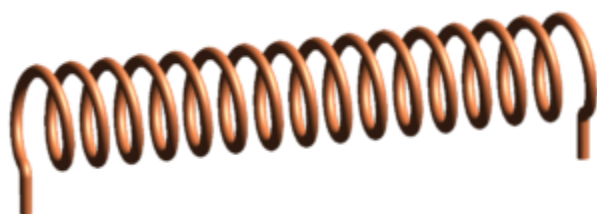
Solution: We know that The Restoring torque is  $\tau = C\theta$ . As  $C$  is torsional constant. However the value of  $C$  is very small in Phosphor-bronze wire, a small restoring torque is generated in the wire. That is, in other words, the Phosphor-bronze wire has a small couple per unit twist.

## The Solenoid and the Toroid

When a charge is lazy enough and sits idle, it is surrounded by an [electric field](#). As it is an electric charge this would make some sense to you. But once that charge gets enthusiastic and starts running around, suddenly it produces a [magnetic field](#). This might strike you as odd, doesn't it? Trust me you aren't the only one! As physicists figured out later, both fields are part of the same force of nature: Electromagnetism! A solenoid uses the concept of [electromagnetism](#).

## Solenoid

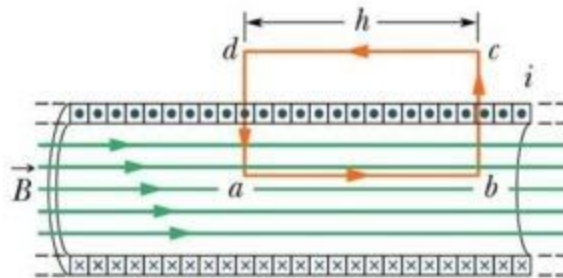
The **solenoid** is a coil of wire that acts like an electromagnet when a flow of electricity passes through it. Electromagnetic solenoids find uses all over the world. You can hardly swing a bat without hitting a solenoid. Speakers and microphones both contain solenoids. In fact, a speaker and microphone are pretty much exactly the same things in reverse of each other.



Solenoid

## Amperian Loop to Determine the Magnetic Field

## Magnetic field of a solenoid



Consider an Amperian loop with sides  $abcd$  and integrate along each side. In  $bc$  and  $da$ , the field is perpendicular to  $dl$ . Along  $cd$ , the field is zero. Along transverse sections  $bc$  and  $ad$ , the field component is zero. Thus, these two sections make no contribution. Let the field along  $ab$  be  $B$ . Thus, the relevant length of the Amperian loop is,  $L = h$ .

Let  $n$  be the number of turns per unit length, then the total number of turns is  $nh$ . The enclosed current  $I_e = I(nh)$ , where  $I$  is the current in the solenoid.

$$BL = \mu_0 I_e, Bh = \mu_0 I(nh)$$

$$B = \mu_0 nI$$

The direction of the field is given by the right-hand rule. The solenoid is commonly used to obtain a uniform magnetic field.

## A long solenoid

A long solenoid is the one which has a larger length in comparison to the radius. It consists of a long wire wound in the form of a helix where the neighbouring turns are closely spaced. So each turn can be regarded as a **circular** loop. The net magnetic field is the vector sum of the fields due to all the turns. Enamelled wires are used for winding so that turns are insulated from each other.

The magnetic field inside a long solenoid is:  $B = \mu_0 nI$  .....(1)

where  $n$  = number of turns per unit length and  $I$  = current flowing through the solenoid. The magnetic field at a point on one end of the long solenoid is:

$$B = (\mu_0 nI / 2) \text{ .....(2)}$$

## Toroid

A toroid is a coil of insulated or enameled wire wound on a donut-shaped form made of powdered iron. low-level inductors,

power inductors, low-level transformers, current transformers and power transformers are some of the applications of Toroid.



Toroid

A toroid is an endless solenoid in the form of a ring. The magnetic field inside a toroid is given as,

$$B = (\mu_0 NI / 2\pi r) \dots\dots\dots(3)$$

where  $I$  = current flowing through the solenoid. Let  $r$  be the average radius of the toroid and  $n$  be the number of turns per unit length and  $N = 2\pi rn = (\text{average}) \text{ perimeter of the toroid} \times \text{number of turns per unit length}$ . On comparing the two results: for a solenoid and toroid.

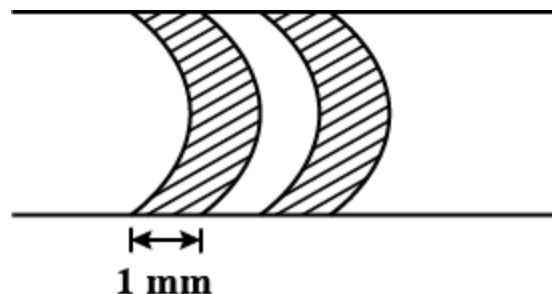
Equations (1) and (3) will give, we get  $B = \mu_0 n I$ , i.e., the result for the solenoid.

## Solved Examples For You

Question: A long solenoid is fabricated to closely winding wire of radius 0.5 mm over a cylindrical frame, so that the successive turns nearly touch each other. The magnetic field at the centre of solenoid, if it carries a current of 5 A is?

- A)  $4\pi \times 10^{-2}$  T      B)  $2\pi \times 10^{-3}$  T      C)  $5\pi \times 10^{-4}$  T      D)  $4\pi \times 10^{-5}$  T

Solution:



The turns of wire on the solenoid are shown in figure.

Number of turns in 1

$$1 \text{ mm} = 1, n = 1000 \text{ turns / m}$$

$$\text{Therefore, } B = \mu_0 n I = 4\pi \times 10^{-7} \times 10^3 \times 5 = 2\pi \times 10^{-3} \text{ T}$$

## Torque on Current Loop, Magnetic Dipole

You may see [objects](#), which when applied force to, show motion with certain restrictions. A door attached to a hinge will rotate around it, the cap of a bottle will turn around its threads, and so on. These [motions](#) are Rotational Motions that use the concept of torque.

Without torque, there would be no twists and turns, no spins!

Wouldn't life be boring that way? Torque gives a rotational motion to an object that would otherwise not be possible.

### Torque

Torque( $\tau$ ) is the twisting force that tends to cause [rotation](#). The axis of rotation is the point where the object rotates.



$$\tau = F \times r$$

Where  $F$  – force applied and  $r$  – the distance between the centre of the axis of rotation and to the point where force is applied.

## Torque On Current Loop, Magnetic Dipole

A **magnetic dipole** is the **limit** of either a closed loop of **electric current** or a pair of poles as the dimensions of the **source** are reduced to zero while keeping the magnetic moment constant. Now we shall show that a steady current  $I$  passing through a rectangular loop placed in a uniform magnetic field experiences a torque. It does not experience a net force. This behaviour is similar to the of an electric dipole in a uniform electric field.

A rectangular current carrying coil in the magnetic field

### Case 1

Let's consider a case when the rectangular loop is placed such that the uniform magnetic field  $B$  is the plane of the loop. The field exerts no force on both arms  $PS$  and  $QR$  of the loop. It is perpendicular to the arm  $PQ$  of the loop and exerts a force  $F_1$  on it which is directed into the plane of the loop. Its magnitude is,

$$F_1 = IZB$$

Similarly, it exerts a force  $F_2$  on the arm  $RS$  and  $F_2$  is directed out of the plane of the paper.

$$F_2 = IzB = F_1$$

Therefore, the net force on the loop is zero. As both the forces  $F_1$  and  $F_2$  nullify each other, there is a torque on the loop. Here, we can see that the torque on the loop tends to rotate it in an anti-clockwise direction.

$$\tau = F_1 (y/2) + F_2 (y/2)$$

$$= IzB (y/2) + IzB (y/2)$$

$$= I (y \times z) B$$

$$= IAB \dots\dots(1)$$

where  $A = y \times z$  is the area of the [rectangle](#).

## Case 2

Now let us consider a case when the plane of the loop is not along the [magnetic field](#) but makes an angle with it. And let us consider the angle between the field and the normal to the coil is angle  $\Theta$ .

The forces on both the arms QR and SP are equal, opposite and act along the axis of the coil, which connects the centres of mass of QR and SP. Being collinear along the axis they cancel out each other, resulting in no net force or torque. The forces on arms PQ and RS are  $F_1$  and  $F_2$ . Furthermore, they too are equal and opposite, with magnitude,

$$F_1 = F_2 = I\ell B$$

As they are not collinear it results in a couple. The effect of torque is, however, less than the earlier case when the plane of the loop was along the magnetic field. The magnitude of the torque on the loop is,

$$\tau = F_1 (y/2) \sin\theta + F_2 (y/2) \sin\theta$$

$$= I (y \times z) B \sin\theta$$

$$= IAB \sin\theta \dots\dots(2)$$

So, the torques in equations (1) and (2) can be expressed as the vector product of the magnetic moment of the coil and the magnetic field.

Therefore, we can define the magnetic moment of the current loop as,

$$\mathbf{m} = IA$$

where  $A$  is the direction of the area vector. The angle between  $\mathbf{m}$  and  $\mathbf{B}$  is  $\theta$ , the equations (1) and (2) can be expressed by one expression

$$\tau = \mathbf{m} \times \mathbf{B}$$

where  $\mathbf{m}$  is the magnetic moment and  $\mathbf{B}$  is the uniform magnetic field.

Learn more about [Magnetic Force and Magnetic Current](#).

## Solved Examples for You

Question: A pole of pole strength  $80 \text{ Am}$  is placed at a point at a distance  $20\text{cm}$  on the equatorial line from the centre of a short magnet of magnetic moment  $20\text{Am}^2$ . Therefore the force experienced by it is

- A)  $8 \times 10^{-2} \text{ N}$     B)  $2 \times 10^{-2} \text{ N}$     C)  $16 \times 10^{-2} \text{ N}$     D)  $64 \times 10^{-2} \text{ N}$

Solution:  $p = 80\text{Am}$ ,  $d = 20\text{cm}$ ,  $m = 20\text{Am}^2$

$$B_2 = \mu_0 m / 4\pi d^3$$

$$= 4\pi \times 10^{-7} \times 20 / 4\pi (0.2)^3$$

$$B = 2.5 \times 10^{-4} \text{ T}$$

$$F = PB$$

$$= 80 \times 2.5 \times 10^{-4}$$

$$F = 0.02 \text{ N}$$