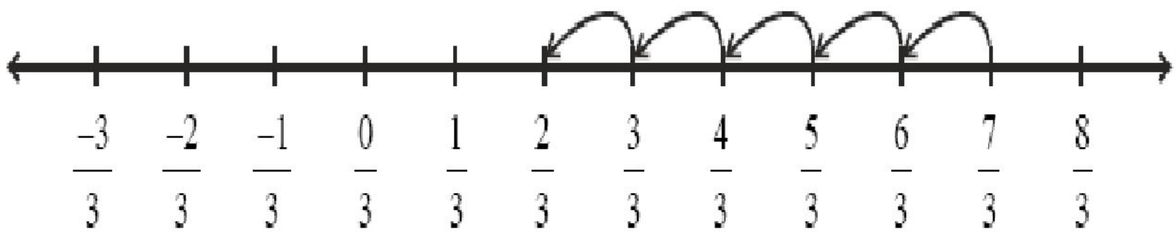
Rational Numbers

What is a Rational Number? When someone asks you about your age, you may say you are 15 years old. The date, the number of pages in a book, the fingers on your hand. What numbers are these? These numbers are something known as rational numbers. Let us study in detail about rational numbers and their properties.

What is a Rational Number?

We already know about some types of numbers. The numbers that we are familiar with are natural numbers, whole numbers, and integers. Natural numbers are the ones that begin with 1 and goes on endlessly up to plus infinity. If we include 0 in these sets of numbers, then these numbers become whole numbers.

Now in these sets, if we also include the negative numbers, then we call it as integers. So all the numbers that we see collectively on the number line are called integers. But what is a rational number?



A rational number is a number that can be written in the form of a numerator upon a denominator. Here the denominator should not be equal to 0. The numerator and the denominator will be integers. A rational number is of the form

p

q

p = numerator, q= denominator, where p and q are integers and q ≠0

Examples:

3

5

,

−3

10

,

11

−15

. Here we can see that all the numerators and denominators are integers and even the denominators should be non-zero.

Positive and Negative Rational Numbers

Any rational number can be called as the positive rational number if both the numerator and denominator have like signs. A rational number which has either the numerator negative or the denominator negative is called the negative rational number.

Identify the Rational Numbers

- **2**
- **7**
- **: Here 2 is an integer, 7 is an integer so yes it is a rational number.**
- **0**
- **0**

- : Here there is 0 in the denominator too. So it is not a rational number.
- -9: Here -9 can be written
- −9
- 1
- . So it is a rational number.
- 0: 0 is a rational number.

Properties of Rational Number

1. ***A rational number remains unchanged when a non zero integer m is multiplied to both numerator and denominator.***

p×m

q×m

Suppose we take the number

2

5

and multiply both numerator and denominator by 3 then,

2×3

5×3

the result that we get is

6

15

. Now this

6

15

is the standard form. If we express it in its simplest form we get it as

2

5

.

2. *A rational number remains unchanged when a non zero same integer m is divided to both numerator and denominator.*

p÷m

q÷m

Suppose we take the number

6

15

and divide both numerator and denominator by 3 then,

6÷3

15÷3

the result that we get is

2

5

.

Standard Form of Rational Number

Now

24

36

is a rational number. But when this number is expressed in its simplest form, it is

2

3

. A rational number is in its standard form if it has no common factors other than 1 between the numerator and denominator and the denominator is positive.

Solved Examples for You

Question: What fraction lies exactly halfway between

2

3

and

3

4

?

A. 3

B. 5
C.

D. 5
E. 6
F.

G. 7
H. 12
I.

J. 9
K. 16
L.

M. 17
N. 4
O.

Solution: The correct option is “E”. Comsider 3 × 4 = 12. So,

2

3

=

8

12

3

4

=

9

12

Multiplying the numerator and denominator by 2

16

24

=

18

24

The midpint is

17

24

Question: If we divide a positive integer by another positive integer, what is the resulting number?

A. **Always a natural number**
B. **Always an integer**
C. **A rational number**
D. **An irrational number**

Solution: The correct option is "C". If we divide a positive integer by another positive integer, the resulting number is always a rational number. Though it can be a natural number and an integer only if the denominator is 1.

Irrational Numbers

You must have studied circle in your lower classes. Well does the topic circle make any sense without π? Absolutely not! Also, the Euler's Number is used extensively in logarithms and algebra.

Well, what are this π and e? Yes, they are known as the irrational numbers. Geometrical calculations involve the irrational numbers. So let us study irrational numbers in detail.

Irrational Numbers

Before studying the irrational numbers, let us define the rational numbers. A rational number is of the form

p

q

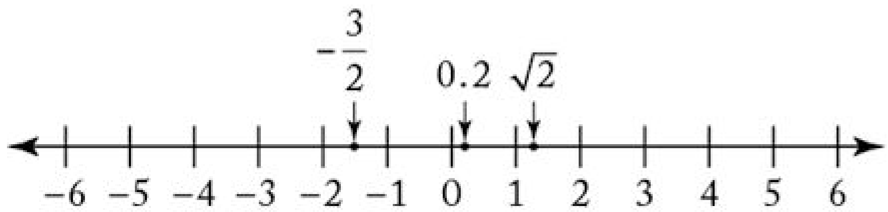
, p = numerator, q= denominator, where p and q are integers and q ≠0.

So irrational number is a number that is not rational that means it is a number that cannot be written in the form

p

q

. It cannot be written as the ratio of two integers. Representation of irrational numbers on a number line



Here the irrational number is √2. So if we calculate the value of √*2,* we get √*2* = 1.14121356230951….. the numbers go on into infinity and do not ever repeat, and they do not ever terminate. It cannot be written in p /q form where q is not equal to zero. The value that we get is actually non terminating. Also, there is no pattern in the digits after the decimal. These kinds of numbers are called irrational numbers.

Now consider √*3.* On calculating we get √*3* = 1.732050807. So the pattern that we get is non terminating and non-recurring. So √*3* is also an irrational number. But in the case of √9, √9 = 3 this is a rational number

The square root of any perfect square will always be a rational number and the square root of any number which is not a perfect square will always be an irrational number. Irrational numbers

have a decimal expansion that never ends and does not repeat. The most famous irrational number is,

Pi = 3.14…..

Pi is used to calculate the ratio of the circumference of a circle to the diameter of that same circle. It has been calculated to over a quadrillion decimal places, but no pattern has ever been found and so it is an irrational number. The few digits of this pattern look like 3.14159265358979323846264338327950. Few examples of irrational numbers are π, e, φ

- e = This is the Euler's number and also an irrational number. The first few digits look like 2.7182818284590452353602874713527…
- φ = This is an irrational number. The first few digits look like 1.61803398874989484820…

Properties of Irrational Numbers

Addition and Subtraction of Irrational numbers

1. ***The result of an addition of irrational numbers need not be an irrational number***

$(2 + \sqrt{3}) + (4 - \sqrt{3}) = 2 + \sqrt{3} + 4 - \sqrt{3} = 6$. Here 6 is a rational number. So the result of adding two irrational numbers is not an irrational number.

2. ***The result of Subtraction of irrational number need not be an irrational number***

$(5+ \sqrt{2}) + (3 + \sqrt{2}) = 5+ \sqrt{2} + 3 + \sqrt{2} = 2$. Here 2 is a rational number.

Multiplication and Division of Irrational numbers

1. ***The product of two irrational numbers can be rational or irrational number.***

$\sqrt{2} \times \sqrt{3}= 6$. Here the result is a rational number.

2. ***The result of the division of two irrational numbers can be rational or irrational number.***

$\sqrt{2} \div \sqrt{3} = \frac{\sqrt{2}}{\sqrt{3}}$. Here the result is an irrational number.

Terminating and Non-terminating Decimals

Decimal numbers with the finite number of digits are called as terminating decimals while decimals with the infinite number of digits are called as non-terminating decimals. The number 0.34 is a terminating decimal, while 0.999… a non-terminating decimal. The symbol… means that the 9 extend indefinitely.

Non-Terminating Recurring Decimals

While expressing a fraction in the decimal form, when we perform division we get some remainder. If the division process does not end means we do not get remainder equal to zero then such decimal is known as non-terminating decimal. In some cases, a digit or a block of digits repeats itself in the decimal part, then the decimal is *non-terminating recurring decimal.* For eg:- 1.666…, 0.141414…

Non-Terminating and Non-recurring Decimals

While expressing a fraction in the decimal form, when we perform division we get some remainder.

If the division process does not end means if we do not get the remainder equal to zero then such decimal is known as non-terminating decimal.

And if a digit or a block of digits does not repeat itself in the decimal part, such decimals are called non-terminating and non-recurring decimals. For eg:- 1.41421356

Conversion of Fractions to Recurring Decimals

$$
\begin{array}{r|l}
 & \ \ 0.181 \\
\hline
11 & \ \ 2.000 \\
 & -11 \\
\cline{2-2}
 & \quad\ \ 90 \\
 & \quad -88 \\
\cline{2-2}
 & \qquad\ \ 20 \\
 & \qquad -11 \\
\cline{2-2}
 & \qquad\quad\ 9
\end{array}
$$

Converting fractions to decimals is the same as dividing two whole numbers. For example, convert

2

11

to recurring decimals.

2

11

= 0.1818…never ends but repeat the digits 18. Hence,

2

11

is expressed in recurring decimal as 0.181818…

Solved Questions for You

Question: State whether the following statements are true or false.

√n is not irrational if n is a perfect square.

- **A. True**
- **B. False**

Solution: The correct option is "A". √4 = 2 where 2 is a rational number. Here n is perfect square the √n is the rational number. √5 = 2.236..is not rational number. But it is an irrational number. Here n is not a perfect square. √n is an irrational number. So √n is not the irrational number if n is a perfect square.

Question: Number of integers lying between 1 to 102 which are divisible by all √2, √3, √5 is

A. 16
B. 17
C. 15
D. 0

Solution: The correct option is "D". For a number to be divisible by √2, it must be an irrational number. An integer is not an irrational number. So there are no numbers between 1 to 102 which all √2, √3, √5.

Irrational Numbers Between Two Numbers

What is an irrational number? Imagine a square having side 1. The diagonal of that square is exactly the square root of two, which is an irrational number. π, e, √3 are examples of irrational numbers. Let's study what is an irrational number between any two numbers.

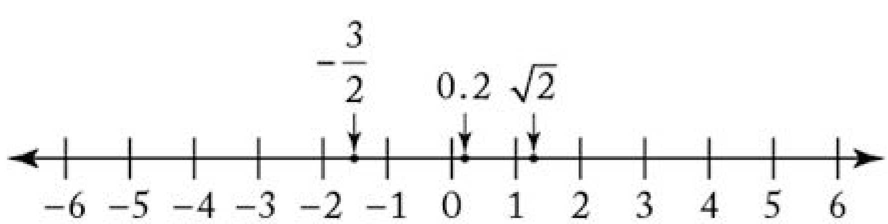
What is an Irrational Number?

An irrational number is a number that is not rational that means it is a number that cannot be written in the form

p

q

. It cannot be written as the ratio of two integers. Representation of irrational numbers on a number line. From the below figure, we can see the irrational number is √2



Irrational Number between Two Rational Numbers

Suppose we have two rational numbers a and b, then the irrational numbers between those two will be, $\sqrt{ab}$. Now let us find two irrational numbers between two given rational numbers.

1. Find an irrational number between two rational numbers $2 - \sqrt{3}$ and $5 - \sqrt{3}$

Let x be the irrational number between two rational numbers $2 - \sqrt{3}$ and $5 - \sqrt{3}$. Then we get,

$2 - \sqrt{3} < x < 5 - \sqrt{3}$

$\Rightarrow 2 < x + < \sqrt{3} < 5$

We see that $x + \sqrt{3}$ is an irrational number between $2 - \sqrt{3}$ and $5 - \sqrt{3}$ where $2 - \sqrt{3} < x < 5 - \sqrt{3}$.

2. Find two irrational numbers between two given rational numbers.

Now let us take any two numbers, say a and b. Let x be any number between a and b. Then,

We have $a < x < b$….. let this be equation (1)

Now, subtract $\sqrt{2}$ from both the sides of equation (1)

So, $a - \sqrt{2} < x < b - \sqrt{2}$……equation (2)

$= a < x + \sqrt{2} < b$

Addition of irrational number with any number results into an irrational number. So, $x + \sqrt{2}$ is an irrational number which exists between two rational numbers a and b.

Irrational Number between Two Irrational Numbers

The easiest way to find the number of two rational numbers is to square both the irrational numbers and take the square root of their average. If the square root is irrational, then we get the number we want. If you do not the number you are looking for, then repeat the procedure using one of the original numbers and the newly generated number.

An irrational number between any two irrational numbers a and b is given by √ab. For example,

1. Find the rational numbers between √2 and √3

Let us first find the difference between √2 and √3. Since the difference lies between

3

10

and

1

3

. There exist an integer between 4√2 and 4√3 that is 6, such that

6

4

=

3

2

 is between √2 and √3. So now we can find other rationals by taking another multiple than 4.

2. We can also find many rationals between any two irrational numbers.

Let us take two irrational numbers a and b. To find the difference between a and b that is b – a, take n ∈ N and n > 1. Now, there exists some integer m between na and nb. Then,

m

n

 is an irrational number between a and b.

Rational Number between Two Rational Numbers

If m and n are the two ration numbers such that m < n then,

1

2

(m + n) is the rational number between m and n. Let us see common denominator method to find the rational number between two rational numbers. Rational numbers between two rational numbers can be found out by using common denominator method. For example,

Let us assume two rational numbers as

−3

2

and

5

3

−3

2

=

−3×3

2×3

=

−9

6

5

3

=

5×2

3×2

=

10

6

Rational numbers between these numbers are

−8

6

,

−7

6

,…,

9

6

Solved Examples for You

Question: Which of the following irrational numbers lies between

3

5

and

9

10

A.

B. √80
C. 10
D.

E. √85
F. 10
G.

H. √82
I. 10
J.

K. √83
L. 10
M.

Solution: A. √36 < √80 < √81. On dividing with 10 we get,

6

10

<

√80

10

<

9

Question: How many irrational numbers lie between $\sqrt{2}$ and $\sqrt{3}$?

A. **One**

B. **Zero**

C. **Ten**

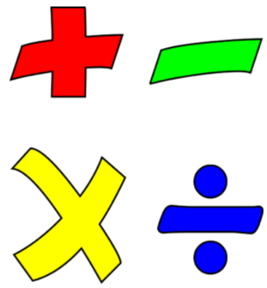
D. **Infinite**

Solution: D. Infinite irrational numbers lie between $\sqrt{2}$ and $\sqrt{3}$. For example, $\sqrt{2.1}$, $\sqrt{2.11}$, $\sqrt{2.101}$ and so on.

Operations on Irrational Numbers

Every one of you is already aware of Rational Numbers and Irrational Numbers. Do you know there are some operations that you can carry out with these real numbers? Let us now study in detail about the operations such as addition, multiplication, subtraction, and division of the rational and irrational numbers.

Rational and Irrational Numbers



An irrational number is a number that is not rational that means it is a number that cannot be written in the form

p

q

. An irrational Number is a number on the Real number line that cannot be written as the ratio of two integers. They cannot be expressed as terminating or repeating decimals. For example.

- **Pi = 3.14…..It continues forever and never repeats. The few digits of this pattern look like 3.1415926535897932384626433832795**
- **√*3* = 1.732050807. So one of the most important thing about irrational numbers is that it never repeats and never terminates.**

Operations on Irrational Numbers

Let us now see the operations of the irrational numbers and the pattern they follow.

Addition of the Irrational Numbers

Irrational Number + Irrational Number = May or may not be an Irrational Number

Example: $\sqrt{2} = 1.414\ldots$, $\sqrt{3} = 1.732\ldots$, $\sqrt{5} = 2.236\ldots$

Let us add these irrational numbers $\sqrt{2} + \sqrt{3}$

$= 1.414\ldots + 1.732\ldots$

$= 3.146\ldots\ldots$

We can see the pattern which we get from adding these numbers is non-repeating and non terminating. So this makes the entire number an irrational number. This is not true in all cases. Let us see another example.

$(5 - \sqrt{2}) + \sqrt{2}$

= 5 – √2 + √2 = 5

We know 5 is definitely an irrational number.

You can see that in the first example when we add two irrational numbers, the result is an irrational number. But in the second example, the addition of two irrational numbers gives us a rational number. Because of this, we say that the addition of two irrational numbers may or may not be an irrational number.

Subtraction of the Irrational Numbers

Irrational Number – Irrational Number = May or may not be an Irrational Number

Again we take the same roots as above.

√2 = 1.414… , √3 = 1.732… , √5 = 2.236…

Let us subtract these irrational numbers

√3 – √2

= 1.732… – 1.414… = 0.318…

Again we see the pattern which we get from subtracting these numbers is non-repeating and non terminating. So this makes the entire number an irrational number. But,

(5 + √2) – √2

So we get = 5 + √2 – √2 = 5

5 is an irrational number. So this example makes it clear that subtraction of two irrational numbers may or may not be an irrational number.

Multiplication of the Irrational Numbers

Irrational Number × Irrational Number = May or may not be an Irrational Number

√2 = 1.414… , √3 = 1.732… , √5 = 2.236…

Let us multiply these irrational numbers

$\sqrt{2} – \sqrt{5}$

= 1.414… × 2.236… = 3.162….

So again this number is non-repeating and non terminating. So this makes the entire number an irrational number. Let us see another example

(5 $\sqrt{3}$) × $\sqrt{3}$

= 5 × $\sqrt{3}$ × $\sqrt{3}$ = 15

Here 15 clearly is a rational number. Because of this, we say that the multiplication of two irrational numbers may or may not be an irrational number.

Division of the Irrational Numbers

Irrational Number / Irrational Number = May or may not be an Irrational Number

$\sqrt{2}$ = 1.414… , $\sqrt{3}$ = 1.732… , $\sqrt{5}$ = 2.236…

$\sqrt{2}$

$\sqrt{3}$

=

1.732…

1.414…

= 1.2234…

We see the pattern which we get from dividing these numbers is non-repeating and non terminating. So this makes the entire number an irrational number. But,

$5\sqrt{5}$

$\sqrt{5}$

= 5

Here 5 is a rational number. Because of this, we say that when we divide two irrational numbers we may or may not get an irrational number.

Solved Examples for You

Question: (16) − (14√2)

A. 2√2

B. 2

C. 30√2

D. 16 – 14√2

Solution: The correct option is "D". (16) − (14√2) is a rational number and 14√2 is an irrational number. We cannot add rational and irrational numbers directly. So we cannot solve the given numbers directly. Therefore, 16 – 14√2 is the answer.

Question: From the pairs of the number given below, whose product is a Rational and Irrational Numbers?

A. √12, √3

B. √4, √3

C. √10, √3

D. √2, √3

Solution: Products of the numbers are as follows

$\sqrt{12} \times \sqrt{3} = 6 \Rightarrow$ natural number

$\sqrt{4} \times \sqrt{3} = \sqrt{12}$ ⇒irrational number

$\sqrt{10} \times \sqrt{3} = \sqrt{30}$ ⇒irrational number

$\sqrt{2} \times \sqrt{3} = \sqrt{6}$ ⇒irrational number

Laws of Exponents for Real Numbers

Supposed you are asked to calculate or multiply large numbers and you don't have an electronic calculator. What if you get the wrong answer? How can we make this calculation easier? So let us try to understand some of the exponent rules for real numbers.

Laws of Exponents or Exponent Rules

What is meant by an exponent? You must have come across the expression 3^2. Here 3 is the base and 2 is the exponent. Exponents are also called Powers or Indices. The exponent of a number tells how many times to use the number in a multiplication. Let us study the laws of exponent. It is very important to understand how the laws of exponents laws are formulated.

(Source: math warehouse)

1. Product law

According to the product law of exponents when multiplying two numbers that have the same base then we can add the exponents

$$a^m \times a^n = a^{m+n}$$

where a, m and n all are natural numbers. Here the base should be the same in both the quantities. For example,

- $2^3 \times 2^4 = 2^7$
- $2^{2/3} \times 2^{1/5} = 2^{2/3 + 1/5} = 2^{(10+3)/15}$ **. We get,** $= 2^{12/15}$
- $(-6)^3 \times (-6)^2 = (-6)^{3+2} = (-6)^5$

2. Quotient Law

According to the quotient law of exponents, we can divide two numbers with the same base by subtracting the exponents. In

order to divide two exponents that have the same base, subtract the power in the denominator from the power in the numerator.

$$a^m \div a^n = a^{m-n}$$

where a, m and n all are natural numbers. Here the base should be the same in both the quantities. For example,

- $2^5 \div 2^3 = 2^2$
- $p^6 \div p^2 = p^{6-2} = p^4$

3. Power Law

According to the power law of exponents if a number raise a power to a power, just multiply the exponents

$$(a^m)^n = a^{m\times n}$$

Here there is one base a and two powers m and n. For example, $(5^3)^2 = 5^{3\times 2} = 5^6$

Important Points to Remember on Exponent Rules

- **1**

- a
- n
- = a^{-n}. **A non zero base raised to a negative exponent is equal to the reciprocal of the base raised to the positive exponent or**
- 1
- a
- –
- n
- = a^{+n}
- **$a^0 = 1$. This says that anything raised to the zero power is 1. For example, $5^0 = 1$, $(1000)^0 = 1$**
- **$a^1 = a$**

Power of Product

The power of product rule states that: $(ab)^m = a^m \times b^m$, a and b are positive real numbers and m is the rational number. For example, $(2 \times 5)^{10} = 2^{10} \times 5^{10}$

Power of Quotient

The power of product rule states that:

a

b

^n =

a

n

b

n

Or,

2

5

^12 =

2

1

2

5

1

Solved Questions on Exponent Rules

Q1. If $12^5 = 3^t \times 4^t$ calculate the value of t.

A. 2.5

B. 5

C. 10

D. 20

E. 40

Solution: The correct option is "B". Given: $12^5 = 3^t \times 4^t$. If two terms have the same power and they are multiplied then the power can be taken as common = $12^5 = (3 \times 4)^t = 12^5 = 12^t$. If two terms have the same base, then we can equate their powers.

Q2. Which of the following expresses the power law of exponents?

A. $a^m \times a^n = a^{m+n}$

B. $a^m \times b^n = (ab)^m$

C. $a^0 = 1$

D. $(a^m)^n = a^{mn}$

Solution: The correct answer is “D”. The power law states that to raise a power to a power, just multiply the exponents. Therefore, $(a^m)^n = a^{m \times n} = a^{mn}$. So, D is the correct option.