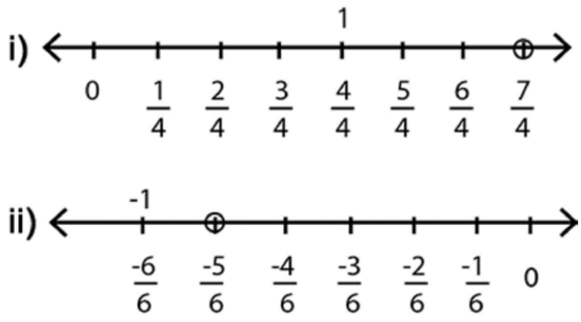
#463115

Represent these numbers on the number line.

(i) $\frac{7}{4}$ (ii) $\frac{-5}{6}$

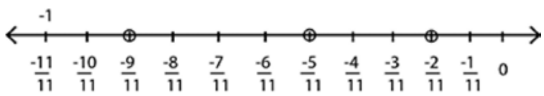
Solution



#463116

Represent $\frac{-2}{11}, \frac{-5}{11}, \frac{-9}{11}$ on the number line.

Solution



#463117

Write five rational numbers which are smaller than 2.

Solution

We can write 2 in the form of $\frac{14}{7}$.

Five rational numbers $= \frac{13}{7}, \frac{12}{7}, \frac{11}{7}, \frac{10}{7}, \frac{9}{7}$

There are multiple solutions for this, as 2 can be written in various forms like $\frac{10}{5}, \frac{4}{2}$, ... and so on.

#463118

Find ten rational numbers between $\frac{-2}{5}$ and $\frac{1}{2}$.

Solution

$\frac{-2}{5}$ and $\frac{1}{2} \rightarrow \frac{-2}{5} = \frac{-8}{20}$ and $\frac{1}{2} = \frac{10}{20}$

$\therefore$ Ten rational numbers are

$\frac{-7}{20}, \frac{-6}{20}, \frac{-5}{20}, \frac{-4}{20}, \frac{-3}{20}, \frac{-2}{20}, \frac{-1}{20}, 0, \frac{1}{20}, \frac{2}{20}$

This question can have different multiple solutions too.

#463119

Find five rational numbers between.

(i) $\frac{2}{3}$ and $\frac{4}{5}$

(ii) $\frac{-3}{2}$ and $\frac{5}{3}$

(iii) $\frac{1}{4}$ and $\frac{1}{2}$

Solution

(i) We can represent $\frac{2}{3}$ as $\frac{30}{45}$ and $\frac{4}{5}$ as $\frac{36}{45}$ respectively.

Therefore;

5 rational numbers $= \frac{31}{45}, \frac{32}{45}, \frac{33}{45}, \frac{34}{45}, \frac{35}{45}$

(ii) We can represent $\frac{-3}{2}$ as $\frac{-9}{6}$ and $\frac{5}{3}$ as $\frac{10}{6}$

Therefore,

5 rational numbers are $\frac{-8}{6}, \frac{-7}{6}, -1, \frac{-5}{6}, \frac{-4}{6}$

(iii) We can represent $\frac{1}{4}$ as $\frac{8}{32}$ and $\frac{1}{2}$ as $\frac{16}{32}$ respectively.

Therefore,

5 rational numbers are $\frac{9}{32}, \frac{10}{32}, \frac{11}{32}, \frac{12}{32}, \frac{13}{32}$

This question can have multiple solutions.

#463120

Write five rational numbers greater than -2.

Solution

We can represent -2 as $\frac{-14}{7}$.

Five rational numbers are

$\frac{-13}{7}, \frac{-12}{7}, \frac{-11}{7}, \frac{-10}{7}, \frac{-9}{7}$

#463521

Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Solution

We know that, rational number is a number which can be written in the form of $\frac{p}{q}$, where q is not equal to zero.

Thus, yes, zero is a rational number, as it can be written in the form of $\frac{p}{q}$.

Since, 0 (zero) is an integer thus, zero can be written in the form of $\frac{p}{q}$

Example : $\frac{0}{1}, \frac{0}{2}, \frac{0}{3}$, etc. where 0 (zero) is an integer..

#463523

Find five rational numbers between 3 and 4.

Solution

Rational number is any number that can express in the form of $\frac{p}{q}$ of two integers, where 'q' cannot be zero.

i) First rational number between 3 and 4 can be calculated by finding average between them, which is

$$\frac{3+4}{2} = \frac{7}{2}$$

Now, we have three numbers i.e. 3, $\frac{7}{2}$ and 4, so other remaining rational numbers can be calculated by taking average between 3 and $\frac{7}{2}$, and between $\frac{7}{2}$ and 4.

ii) Second rational number between 3 and $\frac{7}{2}$ can be calculated by finding average between them.

$$\frac{\frac{7}{2}+3}{2} = \frac{\frac{7+6}{2}}{2} = \frac{13}{4}$$

iii) The third rational number between 4 and $\frac{7}{2}$ can be calculated by finding the average between them.

$$\frac{\frac{7}{2}+4}{2} = \frac{\frac{7+8}{2}}{2} = \frac{15}{4}$$

iv) Similarly fourth rational number between 3 and $\frac{13}{4}$ can be calculated by finding the average between them.

$$\frac{\frac{13}{4}+3}{2} = \frac{\frac{13+12}{4}}{2} = \frac{25}{8}$$

v) Similarly, fifth rational number between 4 and $\frac{13}{4}$ can be calculated by finding average between them.

$$\frac{\frac{13}{4}+4}{2} = \frac{\frac{13+16}{4}}{2} = \frac{29}{8}$$

Then the rational numbers between 3 and 4 are

$$\frac{7}{2}, \frac{13}{4}, \frac{15}{4}, \frac{25}{8}, \frac{29}{8}$$

#463526

Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Solution

Rational number is any number that can express in the form of $\frac{p}{q}$ of two integers, where 'q' cannot be zero, so

(i) The rational number between $\frac{3}{5}$ and $\frac{4}{5}$ is average of $\frac{3}{5}$ and $\frac{4}{5}$

$$\frac{1}{2}\left(\frac{3}{5}+\frac{4}{5}\right)=\frac{1}{2}\left(\frac{3+4}{5}\right)=\frac{7}{10}$$

(ii) The second rational number between $\frac{3}{5}$ and $\frac{4}{5}$ can be calculated bye calculating the average of $\frac{3}{5}$ and $\frac{7}{10}$

$$\frac{1}{2}\left(\frac{3}{5}+\frac{7}{10}\right)=\frac{1}{2}\left(\frac{6+7}{10}\right)=\frac{13}{20}$$

(iii) The third rational number between $\frac{3}{5}$ and $\frac{4}{5}$ can be calculated by calculating the average of $\frac{7}{10}$ and $\frac{4}{5}$

$$\frac{1}{2}\left(\frac{7}{10}+\frac{4}{5}\right)=\frac{1}{2}\left(\frac{7+8}{10}\right)=\frac{15}{20}$$

(iv) The fourth rational number between $\frac{3}{5}$ and $\frac{4}{5}$ can be calculated by calculating the average of $\frac{3}{5}$ and $\frac{13}{20}$

$$\frac{1}{2}\left(\frac{3}{5}+\frac{13}{20}\right)=\frac{1}{2}\left(\frac{12+13}{20}\right)=\frac{25}{40}$$

(v) The fifth rational number between $\frac{3}{5}$ and $\frac{4}{5}$ can be calculated by calculating the average of $\frac{13}{20}$ and $\frac{4}{5}$

$$\frac{1}{2}\left(\frac{13}{20}+\frac{4}{5}\right)=\frac{1}{2}\left(\frac{13+16}{20}\right)=\frac{29}{40}$$

Then five rational are $\frac{7}{10},\frac{13}{20},\frac{15}{20},\frac{27}{40}$ and $\frac{29}{40}$

#463531

State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Solution

(i) True every natural is a whole number because natural number start from 1 and whole number start from 0, so every natural number is a whole number.

(ii) True because an integer is a whole number (not a fractional number) that can be positive or zero.

(iii) False a rational number is any number that can be expressed as the quotient of two integers.

#463534

State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

Solution

(i) True: Real numbers are any number which can we think.

Thus, every irrational number is a real number.

(ii) False: A number line may have negative or positive number.

Since, no negative can be the square root of a natural number, thus every point the the number line cannot be in the form of $\sqrt{m}$, where m is a natural number.

(iii) False: All numbers are real number and non terminating numbers are irrational number.

For example $2, 3, 4,$ etc. are some example of real numbers and these are not irrational.

#463535

Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Solution

No , for example $\sqrt{4} = 2$ is a rational number.

For example, $\sqrt{9} = 3, \sqrt{16} = 4$ is the rational numbers

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#463537

Show how $\sqrt{5}$ can be represented on the number line.

Solution

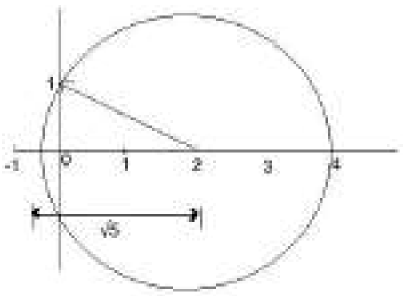
$\sqrt{5} = \sqrt{4+1}$

Here 4 and 1 both are perfect square.

Number as $\sqrt{4} = 2$ and $\sqrt{1} = 1$

So draw a right angle triangle with side 2 and 1.

And according to Pythagoras theorem will be $2^2 + 1^2 = 5$



#463542

Write the following in decimal form and say what kind of decimal expansion each has:

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$ (iv) $\frac{3}{13}$ (v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

Solution

(1) $\frac{36}{100} = 0.36$ It is a terminating decimal.

(2) $\frac{1}{11} = 0.0909..$ It is non-terminating repeating decimal.

(3) $4\frac{1}{8} = \frac{33}{8} = 4.125$ It is a terminating decimal.

(4) $\frac{3}{13} = 0.\overline{230769}$ It is a non-terminating repeating decimal.

(4) $\frac{2}{11} = 0.\overline{18}$ It is a non-terminating repeating decimal.

(5) $\frac{36}{100} = 0.36$ It is a terminating decimal.

#463544

You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how?

Solution

$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$

$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0.\overline{142857} = 0.\overline{428571}$

$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0.\overline{142857} = 0.\overline{571428}$

$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.\overline{142857} = 0.\overline{714285}$

$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0.\overline{142857} = 0.\overline{857142}$

#463546

Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) $0.\bar{6}$

(ii) $0.4\bar{7}$

(iii) $0.\overline{001}$

Solution

(i) Let $x = 0.666$ ....(1)

Multiply by 10 we get

$10x = 6.666$ ....(2)

Substract (1) from (2), we get

$9x = 6 \Rightarrow x = \frac{6}{9} = \frac{2}{3}$

Here $p = 2$ or $q = 3$.

(ii) Let $x = 0.47777$ ....(1)

Multiply by 10 we get

$10x = 4.7777$ ....(2)

Substract (1) from(2), we get

$9x = 4.3 \Rightarrow x = \frac{4.3}{9} = \frac{43}{90}$

Here $p = 43$ or $q = 90$.

(iii) Let $x = 0.001001001$ ....(1)

Multiply by 1000 we get

$1000x = 1.001001$ ....(2)

Substract (1) from (2), we get

$999x = 1 \Rightarrow x = \frac{1}{999}$

Here $p = 1$ or $q = 999$.

#463548

Express 0.99999..... in the form $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Solution

Let $x = 0.99999\cdots$ ...(1)

Multiply eq (1) by 10 we get

$10x = 9.9999\cdots$ ...(2)

Substract (1) from (2), we get

$\Rightarrow 9x = 9$

$\Rightarrow x = \frac{1}{1} = 1$

Here,

$p = 1$ and $q = 1$

Thus,

$0.9999\cdots = 1$

#463550

What can be the maximum number of digits in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Solution

When 1 is divided by 17, we get

$\frac{1}{17} = 0.\overline{0588235294117647}$

So, the number of digit in the repeating block of digit in the decimal expansion of $\frac{1}{17} = 16$

#463551

Look at several examples of rational numbers in the form $\frac{p}{q}(q \neq 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representaions (expansions). Can you guess what property q must satisfy?

Solution

The property that q must satisfy in order that the rational numbers in the from $\frac{p}{q}$, where p and q are integers with no common factor other than 1, have maintaining decimal representation is prime factorization of q has only powers of 2 or power of 5 or both .

i.e $2^m \times 5^n$, where $m = 1, 2, 3, \cdots$ or $n = 1, 2, 3, \cdots$

#463552

Write three numbers whose decimal expansions are non-terminating non-recurring.

Solution

All irrational numbers has non-terminating and non-recurring decimal expression.

So, $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ will give such expressions.

This is one of the various possible answers. It might have different solution.

#463553

Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Solution

$\frac{5}{7} = 0.714285...... = 0.\overline{714285}$

$\frac{9}{11} = 0.8181...... = 0.\overline{81}$

Then three different irrational number between $\frac{5}{7}$ and $\frac{9}{11}$ can be

0.750750075000075............

0.767076700767.............

0.808008000800008..........

This is one of the various possible answers. It might have different solution.

#463555

Classify the following numbers as rational or irrational:

(i) $\sqrt{23}$

(ii) $\sqrt{225}$

(iii) 0.3796

(iv) 7.478478...

(v) 1.101001000100001...

Solution

Solve the given expressions

(1) $\sqrt{23} = 4.795831523..........$

The decimal expansion is non terminating non recurring.

$\therefore$ $\sqrt{23}$ is a irrational number.

(2)$\sqrt{225} = 15$

$\therefore$ $\sqrt{225}$ is a rational number.

(3) 0.3796 is the decimal expansion is terminating.

So 0.3796 is a rational number.

(4) $7.478478...... = 7.\overline{478}$ is the decimal expansion is non termination and recurring.

7.478478...... is a rational number.

(5) 1.101001000100001...... is the decimal expansion is non termination and non recurring. 71.101001000100001...... is a irrational number.

#463556

Visualise 3.765 on the number line, using successive magnification.

Solution

We know that 3.765 lies between 3 and 4.

So, divide the gap between 3 and 4 into ten parts.

So, we get $3, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9, 4$

Now, if we see closely, we can observe that 3.765 lies between 3.6 and 3.7.

So, we divide the gap between 3.7 and 3.8 into ten parts.

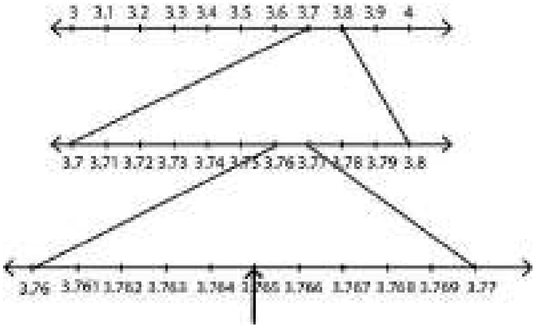
We get $3.7, 3.71, 3.72, 3.73, 3.74, 3.75, 3.76, 3.77, 3.78, 3.79, 3.8$

now if we observe more closely 3.675 lies between 3.67 and 3.68.

Divide the gap between 3.76 and 3.77 into two parts.

So, we get $3.76, 3.765, 3.77$.

Therefore, 3.765 is visualized on number line.



#463557

Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

Solution

We know that 4.2626 lies between 4 and 5.

So, divide the gap between 4 and 5 into ten parts.

So, we get $4, 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 4.9, 5$

Now, if we see closely, we can observe that 4.2626 lies between 4.2 and 4.3.

So, divide the gap between 4.2 and 4.3 into ten parts.

We get $4.2, 4.21, 4.22, 4.23, 4.24, 4.25, 4.26, 4.27, 4.28, 4.29, 4.3$

Now, if we observe more closely 4.2626 lies between 4.26 and 4.27.

Divide the gap between 4.26 and 4.27 into ten parts.

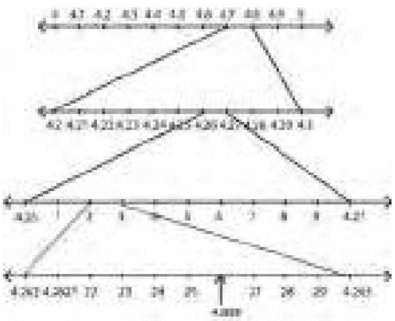
So, we get $4.26, 4.261, 4.262, 4.263, 4.264, 4.265, 4.266, 4.267, 4.268, 4.269, 4.27$.

Now, if we see closely, we can observe that 4.2626 lies between 4.262 and 4.263.

Divide the gap between 4.262 and 4.263 into ten parts.

So, we get $4.262, 4.2621, 4.2622, 4.2623, 4.2624, 4.2625, 4.2626, 4.2627, 4.2628, 4.2629, 4.263$.

Finally, we have spotted 4.2626 on the number line.



#463559

Classify the following numbers as rational or irrational:

(i) $2-\sqrt{5}$ (ii) $(3+\sqrt{23})-\sqrt{23}$ (iii) $\dfrac{2\sqrt{7}}{7\sqrt{7}}$ (iv) $\dfrac{1}{\sqrt{2}}$ (v) 2π

Solution

Only the square roots of the square numbers ; i.e. ,the square roots of the perfect square is rational number

(i) In $2-\sqrt{5}$, the $\sqrt{5}$ is not a rational number, then $2-\sqrt{5}$ is a irrational number

(ii) In $(3+\sqrt{23})-\sqrt{23}=3+\sqrt{23}-\sqrt{23}=3$ is a rational number.

(iii) $\dfrac{2\sqrt{7}}{7\sqrt{7}}=\dfrac{2}{7}$ is a rational number

(iv) $\dfrac{1}{\sqrt{2}}$ the $\sqrt{2}$ is not a rational number, then $\dfrac{1}{\sqrt{2}}$ is irrational number

(v) π is an irrational number. Hence 2π is an itrational number

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#463561

Simplify each of the following expressions:

(i) $(3+\sqrt{3})(2+\sqrt{2})$

(ii) $(3+\sqrt{3})(3-\sqrt{3})$

(iii) $(\sqrt{5}+\sqrt{2})^2$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Solution

We know that, $(A+B)(C+D)=A\times(C+D)+B\times(C+D)$

$=AC+AD+BC+CD$

(i) $(3+\sqrt{3})(2+\sqrt{2})=(6+2\sqrt{3}+3\sqrt{2}+\sqrt{6})$

(ii) $(\sqrt{5}+\sqrt{2})^2=(5+2+2\sqrt{10})=(7+2\sqrt{10})$

(iii) $(3+\sqrt{3})(3-\sqrt{3})=(9-3)=6$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})=(5-2)=3$

#463562

Recall, π is defined as the ratio of the circumference(say c) of a circle to its diameter(say d). That is, $\pi=\dfrac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Solution

π is a ratio of magnitudes,circumference/diameter, but it is not a ratio of integers.

for example, $(\sqrt{5}-1)/2$ is an irrational number but it is expressed in c/d form.

therefore π is an irrational number. if circumference is integer then diameter is not integer and viceversa.

Here,

$\pi = \frac{22}{7}$

Now, it is in the form of $\frac{p}{q}$, which is a rational number, but this is an approximate value of π.

If you divide 22 by 7, the quotient $(3.14\cdots)$ is a non-terminating number i.e. it is irrational.

Approximate fractions include (in order of increasing accuracy)

$\frac{22}{7}, \frac{333}{106}, \frac{355}{113}, \cdots$

If we divide anyone of these we get, the quotient $(3.14\cdots)$ is a non-terminating number. i.e. it is irrational.

So,

There is no contradiction as either c or d irrational and

hence π is a irrational number.

#463563

Represent $\sqrt{9.3}$ on the number line.

Solution

Steps:

1) Draw a line segment AB of length 9.3 units.

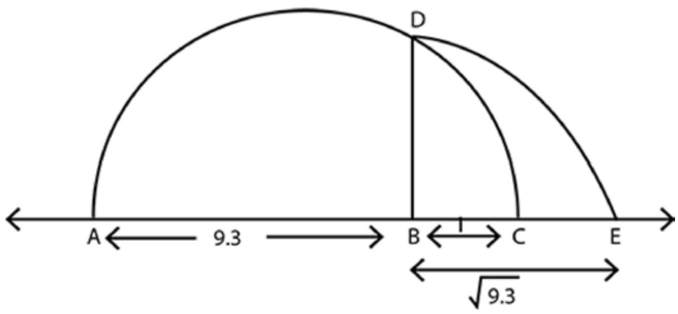
2) Extend the line by 1 unit more such that $BC = 1$ unit .

3) Find the midpoint of AC.

4) Draw a line BD perpendicular to AB and let it intersect the semicircle at point D.

5) Draw an arc DE such that $BE = BD$.

Therefore, BE=$\sqrt{9.3}$ units



#463566

Rationalise the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

(iv) $\frac{1}{\sqrt{7}-2}$

Solution

(1) $\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$ (multiplying and dividing by $\sqrt{7}$)

$= \frac{\sqrt{7}}{7}$

(2) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$ (multiplying and dividing by $\sqrt{7}+\sqrt{6}$)

$= \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$

(3) $\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ (multiplying and dividing by $\sqrt{5}-\sqrt{2}$

$= \frac{\sqrt{5}-\sqrt{2}}{5-2} = \frac{\sqrt{5}-\sqrt{2}}{3}$

(4) $\frac{1}{\sqrt{7}-2} = \frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2}$(multiplying and dividing by $\sqrt{7}+2$)

$= \frac{\sqrt{7}+2}{7-4} = \frac{\sqrt{7}+2}{3}$

#463567

Find:

(i) $64^{\frac{1}{2}}$ (ii) $32^{\frac{1}{5}}$ (iii) $125^{\frac{1}{3}}$

Solution

i) $(64)^{\frac{1}{2}}$

Express 64 in terms of 8

$= (8^2)^{\frac{1}{2}} = 8^{2\times\frac{1}{2}} = 8^1 = 8$

ii) $(32)^{\frac{1}{5}}$

Express 32 in terms of 2

$= (2^5)^{\frac{1}{5}} = 2^{5\times\frac{1}{5}} = 2^1 = 2$

iii) $(125)^{\frac{1}{3}}$

Express 125 in terms of 5

$= (5^3)^{\frac{1}{3}} = 5^{3\times\frac{1}{3}} = 5^1 = 5$

#463569

Find:

(i) $9^{\frac{3}{2}}$ (ii) $32^{\frac{2}{5}}$ (iii) $16^{\frac{3}{4}}$ (iv) $125^{\frac{-1}{3}}$

Solution

(i) $9^{\frac{3}{2}}$

Express 9 in terms of 3

$= (3^2)^{\frac{3}{2}} = 3^{2\times\frac{3}{2}} = 3^3 = 27$

(ii) $32^{\frac{2}{5}}$

Express 32 in terms of 2

$= (2^5)^{\frac{2}{5}} = 2^{5\times\frac{2}{5}} = 2^2 = 4$

(iii) $16^{\frac{3}{4}}$

Express 16 in terms of 4

$= (2^4)^{\frac{3}{4}} = 2^{4\times\frac{3}{4}} = 2^3 = 8$

(iv) $125^{\frac{-1}{3}}$

Express 125 in terms of 5

$= (5^3)^{\frac{-1}{3}} = 5^{3\times\frac{-1}{3}} = 5^{-1} = \dfrac{1}{5}$

#463571

Simplify:

(i) $2^{\frac{3}{2}}2^{\frac{1}{5}}$ (ii) $\left(\dfrac{1}{3^3}\right)^7$ (iii) $\dfrac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ (iv) $7^{\frac{1}{2}}\cdot 8^{\frac{1}{2}}$

Solution

(i) $2^{\frac{3}{2}}.2^{\frac{1}{5}}$

By using property $X^a.\ X^b{=}X^{a+b}$

$= 2^{\frac{3}{2}+\frac{1}{5}} = 2^{\frac{17}{10}}$

(ii) $\left(\dfrac{1}{3^3}\right)^7$

By using property $\left(\dfrac{x}{y}\right)^a = \dfrac{x^a}{y^b}$

$= \dfrac{1}{(3^3)^7}$

$= \dfrac{1}{3^{21}}$ ...... $[\because (x^a)^b = x^{ab}]$

(iii) $\dfrac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

By using property $\dfrac{X^a}{X^b} = X^{a-b}$

$= 11^{\frac{1}{2}-\frac{1}{4}} = 11^{\frac{1}{4}}$

(iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}}$

$= 56^{\frac{1}{2}}$ ...... $[\because x^a y^a = (xy)^a]$

#464961

Prove that $\sqrt{5}$ is irrational.

Solution

Let us assume, to the contrary, that $\sqrt{5}$ is rational.

So, we can find the integers r and s $\neq 0$ such that $\sqrt{5} = \frac{r}{s}$

Suppose r and s have a common factor other than 1. Then, we divide by the common factor to get $\sqrt{5} = \frac{a}{b}$, where a and b are co-prime.

So, $b\sqrt{5} = a$

Squaring on both the sides, we get

$5b^2 = a^2$ ---(1)

Therefore, 5 divides a^2.

Now, according to the theorem of rational number, for any prime number p which divides a^2 then it will divide a also.

That means 5 will divide a. So we can write

$a = 5c$

Subtitute the value of a in equation (1)

$5b^2 = (5c)^2$

$5b^2 = 25c^2$

Divide by 25 we get

$\frac{b^2}{5} = c^2$

Again using the same theorem we get that be will be divide by 5 and we have already get that a is divide by 5 but a and b are co-prime number.

This contradiction has arised because of our incorrect assumption that $\sqrt{5}$ is irrational.

So, we conclude that $\sqrt{5}$ is irrational.

#464968

Prove that $3 + 2\sqrt{5}$ is irrational.

Solution

Let us assume $3 + 2\sqrt{5}$ + is rational.

So we can write this number as

$3 + 2\sqrt{5} = \frac{a}{b}$ ---- (1)

Here a and b are two co-prime number and b is not equal to zero.

Simplify the equation (1) subtract 3 both sides, we get

$2\sqrt{5} = \frac{a}{b} - 3$

$2\sqrt{5} = \frac{a - 3b}{b}$

Now divide by 2 we get

$\sqrt{5} = \frac{a - 3b}{2b}$

Here a and b are integer so $\frac{a - 3b}{2b}$ is a rational number, so $\sqrt{5}$ should be a rational number.

But $\sqrt{5}$ is a irrational number, so it is contradict.

Therefore, $3 + 2\sqrt{5}$ is irrational number.

#464974

Prove that the following are irrational:

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

Solution

(i) $\frac{1}{\sqrt{2}}$

Let us assume $\frac{1}{\sqrt{2}}$ is rational.

So we can write this number as

$\frac{1}{\sqrt{2}} = \frac{a}{b}$ ---- (1)

Here, a and b are two co-prime numbers and b is not equal to zero.

Simplify the equation (1) multiply by $\sqrt{2}$ both sides, we get

$1 = \frac{a\sqrt{2}}{b}$

Now, divide by b, we get

$b = a\sqrt{2}$ or $\frac{b}{a} = \sqrt{2}$

Here, a and b are integers so, $\frac{b}{a}$ is a rational number, so $\sqrt{2}$ should be a rational number.

But $\sqrt{2}$ is a irrational number, so it is contradictory.

Therefore, $\frac{1}{\sqrt{2}}$ is irrational number.

(ii) $7\sqrt{5}$

Let us assume $7\sqrt{5}$ is rational.

So, we can write this number as

$7\sqrt{5} = \frac{a}{b}$ ---- (1)

Here, a and b are two co-prime numbers and b is not equal to zero.

Simplify the equation (1) divide by 7 both sides, we get

$\sqrt{5} = \frac{a}{7b}$

Here, a and b are integers, so $\frac{a}{7b}$ is a rational number, so $\sqrt{5}$ should be a rational number.

But $\sqrt{5}$ is a irrational number, so it is contradictory.

Therefore, $7\sqrt{5}$ is irrational number.

(iii) $6 + \sqrt{2}$

Let us assume $6 + \sqrt{2}$ is rational.

So we can write this number as

$6 + \sqrt{2} = \frac{a}{b}$ ---- (1)

Here, a and b are two co-prime number and b is not equal to zero.

Simplify the equation (1) subtract 6 on both sides, we get

$\sqrt{2} = \frac{a}{b} - 6$

$\sqrt{2} = \frac{a - 6b}{b}$

Here, a and b are integers so, $\frac{a - 6b}{b}$ is a rational number, so $\sqrt{2}$ should be a rational number.

But $\sqrt{2}$ is a irrational number, so it is contradictory.

Therefore, $6 + \sqrt{2}$ is irrational number.

#464986

Write down the decimal expansions of the rational numbers which have terminating decimal expansions.

(i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{15}{1600}$ (iv) $\frac{23}{2^3 5^2}$ (v) $\frac{6}{15}$ (vi) $\frac{35}{50}$

Solution

The division of all the numbers is shown in figure.

i) $\frac{13}{3125}$ = 0.00416

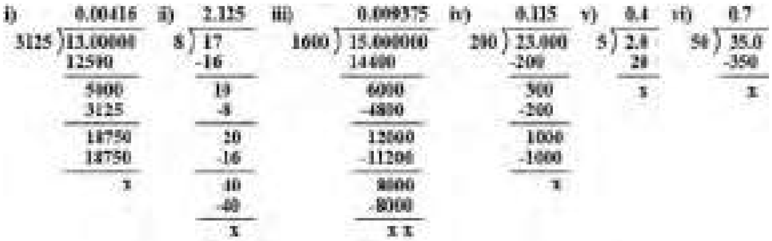
ii) $\frac{17}{8}$ = 2.125

iii) $\frac{15}{1600}$ = 0.009375

iv) $\frac{23}{2^3 5^2}$ = 0.115

v) $\frac{6}{15}$ = 0.4

vi) $\frac{35}{50}$ = 0.70



#465887

Find ten rational numbers between $\frac{3}{5}$ and $\frac{3}{4}$.

Solution

We can represent $\frac{3}{5}$ as $\frac{36}{60}$ and $\frac{3}{4}$ as $\frac{45}{60}$ respectively.

Therefore;

10 rational numbers are

$\frac{37}{60}, \frac{38}{60}, \frac{39}{60}, \frac{40}{60}, \frac{41}{60}, \frac{42}{60}, \frac{43}{60}, \frac{44}{60}, \frac{45}{60}, \frac{46}{60}$

This question might have a different answer. Answer showed here is one of the several possibility.