Download Toppr - India's best learning app for classes 5th to 12th

360° learning with our adaptive platform

**Online Classes**
Learn for free with short videos and live classes

**Adaptive Practice**
Practice smart with questions created for your unique needs

**Mock Tests**
Be exam-ready by solving all India tests and previous years' papers

**Live Doubts**
Chat with tutors and get your doubts resolved instantly, 24x7

**Live Classes**
Learn concepts and get tips from the best teachers with free Live Classes

Learn for free with the best videos

Polish your basics with fun games

Practice smart with personalised questions

Get your doubts cleared instantly, 24x7

Download the app for FREE now

GET A 5-DAY FREE TRIAL

[Android App on Google Play] [Get it on App Store]

9,184,321 Happy Students
492,461,127 Questions Attempted
3,986,828 Tests Taken
8,017,171 Doubts Answered
NCERT Solutions for Class 11 Subjectwise

- Class 11 Mathematics
- Class 11 Physics
- Class 11 Biology
- Class 11 Chemistry
- Class 11 English
- Class 11 Accountancy
- Class 11 Business Study
- Class 11 Economics
A point is on the $x$-axis. What are its $y$-coordinate and $z$-coordinates?

Solution
If a point is on the $x$-axis then only $x$-coordinate will have non-zero constant value and other coordinates will be zero.
Hence, $y$-coordinates and $z$-coordinates are zero.

If a point is in the $yz$-plane, what can you say about its $x$-coordinate?

Solution
If a point is in the $yz$-plane then its $x$-coordinate is zero.

Name the octants in which the following points lie:

$(1, 2, 3), (4, 2, 3), (4, 2, -5), (-4, 2, -5), (-4, 2, 5), (-3, -1, 6), (-2, -4, -7)$

Solution
The $x$-coordinate, $y$-coordinate and $z$-coordinate of point $(1, 2, 3)$ are all positive.
Therefore this point lies in octant I.
The $x$-coordinate, $y$-coordinate and $z$-coordinate of point $(4, 2, 3)$ are positive negative and positive respectively.
Therefore this point lies in octant IV.
The $x$-coordinate, $y$-coordinate and $z$-coordinate of point $(4, 2, -5)$ are positive negative and negative respectively.
Therefore this point lies in octant VIII.
The $x$-coordinate, $y$-coordinate and $z$-coordinate of point $(4, 2, 5)$ are positive positive and negative respectively.
Therefore this point lies in octant V.
The $x$-coordinate, $y$-coordinate and $z$-coordinate of point $(-4, 2, -5)$ are negative positive, and negative respectively.
Therefore this point lies in octant VI.
The $x$-coordinate, $y$-coordinate and $z$-coordinate of point $(-4, 2, 5)$ are negative positive and positive respectively.
Therefore this point lies in octant II.
The $x$-coordinate, $y$-coordinate and $z$-coordinate of point $(-3, -1, 6)$ are negative negative and positive respectively.
Therefore this point lies in octant III.
The $x$-coordinate, $y$-coordinate and $z$-coordinate of point $(-2, -4, -7)$ are positive negative and negative respectively.
Therefore this point lies in octant VIII.

Fill in the blanks:
(i) The $x$-axis and $y$-axis taken together determine a plane known as ________
(ii) The coordinates of points in the $xy$-plane are of the form ________
(iii) Coordinate planes divide the space into ________ octants

Solution
(i) The $x$-axis and $y$-axis taken together determine a plane known as $xy$-plane
(ii) The coordinates of points in the $xy$-plane are of the form $(x, y)$, where $x$ is $x$-coordinate and $y$ is $y$-coordinate
(iii) Coordinate planes divide the space into 8 octants.

Topic: Distance Between Two Points
Find the distance between the following pairs of points:

(i) \((2, 3, 5)\) and \((4, 3, 1)\)
(ii) \((-3, 7, 2)\) and \((2, 4, -1)\)
(iii) \((-1, 3, -4)\) and \((1, -3, 4)\)
(iv) \((2, -1, 3)\) and \((-2, 1, 3)\)

Solution

The distance between points \(P(x_1, y_1, z_1)\) and \(Q(x_2, y_2, z_2)\) is given by

\[d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\]

(i) Distance between points \((2, 3, 5)\) and \((4, 3, 1)\)

\[d = \sqrt{(4 - 2)^2 + (3 - 3)^2 + (1 - 5)^2}\]
\[= \sqrt{4 + 0 + 16}\]
\[= \sqrt{20}\]
\[= 2\sqrt{5}\]

(ii) Distance between points \((-3, 7, 2)\) and \((2, 4, -1)\)

\[d = \sqrt{(2 + 3)^2 + (4 - 7)^2 + (-1 - 2)^2}\]
\[= \sqrt{25 + 9 + 9}\]
\[= \sqrt{43}\]

(iii) Distance between points \((-1, 3, -4)\) and \((1, -3, 4)\)

\[d = \sqrt{(1 + 1)^2 + (-3 - 3)^2 + (-4 + 4)^2}\]
\[= \sqrt{4 + 36 + 0}\]
\[= \sqrt{40}\]
\[= 2\sqrt{10}\]

(iv) Distance between points \((2, -1, 3)\) and \((-2, 1, 3)\)

\[d = \sqrt{(-2 - 2)^2 + (1 + 1)^2 + (3 - 3)^2}\]
\[= \sqrt{16 + 4}\]
\[= \sqrt{20}\]
\[= 2\sqrt{5}\]

#418298

**Topic:** Distance Between Two Points

Verify the following

(i) \((0, 7, -10), (1, 6, -6)\) and \((4, 9, -6)\) are the vertices of an isosceles triangle
(ii) \((0, 7, 10), (-1, 6, 6)\) and \((-4, 9, 6)\) are the vertices of a right angled triangle
(iii) \((-1, 2, 5), (1, -2, 5)\) and \((-4, 7, 8)\) and \((2, -3, 4)\) are the vertices of a parallelogram

Solution
ii) Let point \(0, 7, -10\), \(6, 6, -6\) and \(4, 9, -6\) be denoted by \(A, B\) and \(C\) respectively

\[
AB = \sqrt{(-7)^2 + (6 - 6)^2 + (-6 + 10)^2} \\
= \sqrt{49 + 0 + 16} \\
= \sqrt{65} \\
\Rightarrow AB = 8.06
\]

\[
BC = \sqrt{(4 - 7)^2 + (9 - 6)^2 + (-4 - 6)^2} \\
= \sqrt{9 + 9 + 100} \\
= \sqrt{118} \\
\Rightarrow BC = 10.86
\]

\[
CA = \sqrt{(0 - 4)^2 + (7 - 9)^2 + (10 - 6)^2} \\
= \sqrt{16 + 4 + 16} \\
= \sqrt{36} = 6
\]

Here \(AB = BC \neq CA\)

Thus the given points are the vertices of an isosceles triangle.

iii) Let \((0, 7, 10), (1, 6, 6)\) and \((-4, 9, 6)\) be denoted by \(A, B\) and \(C\) respectively.

\[
AB = \sqrt{(-1)^2 + (6 - 7)^2 + (6 - 10)^2} \\
= \sqrt{1 + 1 + 16} \\
= \sqrt{18} \\
= 3\sqrt{2}
\]

\[
BC = \sqrt{(-4 + 1)^2 + (9 - 6)^2 + (6 - 6)^2} \\
= \sqrt{9 + 9 + 0} \\
= \sqrt{18} \\
\Rightarrow BC = 3\sqrt{2}
\]

\[
CA = \sqrt{(0 + 4)^2 + (7 - 9)^2 + (10 - 6)^2} \\
= \sqrt{16 + 4 + 16} \\
= \sqrt{36}
\]

Now \(AB^2 + BC^2 = (3\sqrt{2})^2 + (3\sqrt{2})^2 = 18 + 18 = 36 = AC^2\)

Therefore by pythagoras theorem \(ABC\) is a right triangle.

Hence the given points are the vertices of a right-angled triangle.

iv) Let \((-1, 2, 1), (1, -2, 5)\), \((-4, 9, 6)\) and \((2, -3, 4)\) be denoted by \(A, B, C\) and \(D\) respectively.

\[
AB = \sqrt{(-1 + 1)^2 + (-2 - 2)^2 + (5 - 5)^2} \\
= \sqrt{16 + 16} \\
= \sqrt{36}
\]

\[
AB = 6
\]

\[
BC = \sqrt{(4 - 1)^2 + (-7 + 2)^2 + (6 - 6)^2} \\
= \sqrt{9 + 25 + 0} \\
= \sqrt{34}
\]

\[
CD = \sqrt{(2 - 4)^2 + (-3 + 7)^2 + (4 - 8)^2} \\
= \sqrt{4 + 16 + 16} \\
= \sqrt{36}
\]

\[
CD = 6
\]

\[
DA = \sqrt{(-1 - 2)^2 + (2 + 3)^2 + (1 - 4)^2} \\
= \sqrt{9 + 25 + 9} \\
= \sqrt{43}
\]

Here \(AB = CD = 6, BC = AD = \sqrt{43}\)

Hence the opposite sides of quadrilateral \(ABCD\) whose vertices are taken in order are equal.

Therefore \(ABCD\) is a parallelogram.

Hence the given points are the vertices of a parallelogram.
#418308
**Topic:** Distance Between Two Points

Find the equation of the set of points which are equidistant from the points $A(1, 2, 3)$ and $B(3, 2, -1)$.

**Solution**

Let $P(x, y, z)$ be the point that is equidistant from points $A(1, 2, 3)$ and $B(3, 2, -1)$.

i.e. $PA = PB$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$

$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$

$\Rightarrow -2x + 6z = -14 = -6x + 2z + 14$

$\Rightarrow -2x - 6z = 6x - 2z = 0$

$\Rightarrow 4x - 8z = 0$

$\Rightarrow x = 2z$

Thus the required equation is $x = 2z$.

#418320
**Topic:** Distance Between Two Points

Find the equation of the set of points $P$, the sum of whose distances from two points $A(4, 0, 0)$ and $B(-4, 0, 0)$ is equal to 10.

**Solution**

Let the coordinates of $P$ be $(x, y, z)$.

The coordinates of points $A$ and $B$ are $(4, 0, 0)$ and $(-4, 0, 0)$ respectively.

It is given that $PA + PB = 10$

$\Rightarrow \sqrt{x^2 - 4x^2 + y^2 + z^2} + \sqrt{x^2 + 4x^2 + y^2 + z^2} = 10$

$\Rightarrow \sqrt{x^2 - 4x^2 + y^2 + z^2} = 10 - \sqrt{x^2 + 4x^2 + y^2 + z^2}$

On squaring both sides, we obtain

$\Rightarrow x^2 + 4x^2 + y^2 + z^2 = 100 - 20\sqrt{x^2 + 4x^2 + y^2 + z^2} = 100 - 20\sqrt{x^2 + 8x + 16 + y^2 + z^2} + x^2 + 8x + 16 + y^2 + z^2$

$\Rightarrow 20\sqrt{x^2 + 8x + 16 + y^2 + z^2} = 100 + 16x$

$\Rightarrow 5\sqrt{x^2 + 8x + 16 + y^2 + z^2} = 25 + 4x$

On squaring both sides again, we obtain

$25(x^2 + 8x + 16 + y^2 + z^2) = 625 + 16x^2 + 200x$

$\Rightarrow 25x^2 + 200x + 400 + 25y^2 + 25z^2 = 625 + 16x^2 + 200x$

$\Rightarrow 9x^2 + 25y^2 + 25z^2 = 225 = 0$

Thus the required equation is $9x^2 + 25y^2 + 25z^2 = 225$.

#418328
**Topic:** Section Formula

Find the coordinates of the point which divides the line segment joining the points $(-2, 3, 5)$ and $(4, -4, 6)$ in the ratio

(i) 2 : 3 internally

(ii) 2 : 3 externally

**Solution**


(i) The coordinates of point R that divides the line segment joining points \( P(x_1, y_1, z_1) \) and \( Q(x_2, y_2, z_2) \) internally in the ratio \( m:n \) are

\[
\left( \frac{m x_1 + n x_2}{m + n}, \frac{m y_1 + n y_2}{m + n}, \frac{m z_1 + n z_2}{m + n} \right)
\]

Let \( R(x, y, z) \) be the point that divides the line segment joining points \(-2, 3, 5\) and \(1, -4, 6\) internally in the ratio 2:3

\[
x = \frac{2(-2) + 3(1)}{2 + 3}, \quad y = \frac{2(3) + 3(-4)}{2 + 3} \quad \text{and} \quad z = \frac{2(5) + 3(6)}{2 + 3}
\]

i.e., \( x = -\frac{4}{5}, \quad y = \frac{1}{5} \quad \text{and} \quad z = \frac{27}{5} \)

Thus the coordinates of the required point are \(\left(-\frac{4}{5}, \frac{1}{5}, \frac{27}{5}\right)\)

(ii) The coordinates of point R that divides the line segment joining points \( P(x_1, y_1, z_1) \) and \( Q(x_2, y_2, z_2) \) externally in the ratio \( m:n \) are

\[
\left( \frac{m x_1 - n x_2}{m - n}, \frac{m y_1 - n y_2}{m - n}, \frac{m z_1 - n z_2}{m - n} \right)
\]

Let \( R(x, y, z) \) be the point that divides the line segment joining points \(-2, 3, 5\) and \(1, -4, 6\) externally in the ratio 2:3

\[
x = \frac{2(1) - 3(-2)}{2 - 3}, \quad y = \frac{2(-4) - 3(3)}{2 - 3} \quad \text{and} \quad z = \frac{2(6) - 3(5)}{2 - 3}
\]

i.e., \( x = -8, \quad y = 17 \quad \text{and} \quad z = 3 \)

Thus the coordinates of the required point are \((-8, 17, 3)\)

---

**#418342**

**Topic:** Section Formula

Given that \( P(3, 2, -4), Q(5, 4, -6) \) and \( R(9, 8, -10) \) are collinear. Find the ratio in which \( Q \) divides \( PR \).

**Solution**

Given points \( P(3, 2, -4), Q(5, 4, -6) \) and \( R(9, 8, -10) \).

Let \( Q \) divides \( PR \) in the ratio \( k:1 \).

So, by section formula, coordinates of \( Q \) are \(\left( \frac{k(9) + 1(3)}{k + 1}, \frac{k(8) + 1(2)}{k + 1}, \frac{k(-10) + 1(-4)}{k + 1} \right)\)

\[
= \left( \frac{9k + 3}{k + 1}, \frac{8k + 2}{k + 1}, \frac{-10k - 4}{k + 1} \right)
\]

But the given coordinates of \( Q \) are \((5, 4, -6)\).

On comparing \(\frac{9k + 3}{k + 1} = 5\)

\(\Rightarrow 9k + 3 = 5k + 5\)

\(\Rightarrow 4k = 2\)

\(\Rightarrow k = 2\)

So, \( Q \) divides \( PR \) in the ratio \(1:2\).

Let point \( Q(5, 4, -6) \) divides the line segment joining points \( P(3, 2, -4) \) and \( R(9, 8, -10) \) in the ratio \( k:1 \).

Thus using section formula,

\[
(5, 4, -6) = \left( \frac{k(9) + 1(3)}{k + 1}, \frac{k(8) + 1(2)}{k + 1}, \frac{k(-10) + 1(-4)}{k + 1} \right)
\]

\[
= \left( \frac{9k + 3}{k + 1}, \frac{8k + 2}{k + 1}, \frac{-10k - 4}{k + 1} \right)
\]

\(\Rightarrow 9k + 3 = 5k + 5\)

\(\Rightarrow 9k + 3 = 5k + 5\)

\(\Rightarrow 4k = 2\)

\(\Rightarrow k = \frac{1}{2}\)

Hence point \( Q \) divides \( PR \) in the ratio 1:2.

---

**#418344**

**Topic:** Section Formula

Given points \( P(3, 2, -4), Q(5, 4, -6) \) and \( R(9, 8, -10) \) are collinear. Find the ratio in which \( Q \) divides \( PR \).

**Solution**

Given points \( P(3, 2, -4), Q(5, 4, -6) \) and \( R(9, 8, -10) \).

Let \( Q \) divides \( PR \) in the ratio \( k:1 \).

So, by section formula, coordinates of \( Q \) are \(\left( \frac{k(9) + 1(3)}{k + 1}, \frac{k(8) + 1(2)}{k + 1}, \frac{k(-10) + 1(-4)}{k + 1} \right)\)

\[
= \left( \frac{9k + 3}{k + 1}, \frac{8k + 2}{k + 1}, \frac{-10k - 4}{k + 1} \right)
\]

But the given coordinates of \( Q \) are \((5, 4, -6)\).

On comparing \(\frac{9k + 3}{k + 1} = 5\)

\(\Rightarrow 9k + 3 = 5k + 5\)

\(\Rightarrow 4k = 2\)

\(\Rightarrow k = 2\)

So, \( Q \) divides \( PR \) in the ratio \(1:2\).

Let point \( Q(5, 4, -6) \) divides the line segment joining points \( P(3, 2, -4) \) and \( R(9, 8, -10) \) in the ratio \( k:1 \).

Thus using section formula,

\[
(5, 4, -6) = \left( \frac{k(9) + 1(3)}{k + 1}, \frac{k(8) + 1(2)}{k + 1}, \frac{k(-10) + 1(-4)}{k + 1} \right)
\]

\(\Rightarrow 9k + 3 = 5k + 5\)

\(\Rightarrow 9k + 3 = 5k + 5\)

\(\Rightarrow 4k = 2\)

\(\Rightarrow k = \frac{1}{2}\)

Hence point \( Q \) divides \( PR \) in the ratio 1:2.
Find the ratio in which the \( yz \)-plane divides the line segment formed by joining the points \((-2, 4, 7)\) and \((3, -5, 8)\).

Solution

Let the \( yz \)-plane divide the line segment joining points \((-2, 4, 7)\) and \((3, -5, 8)\) in the ratio \(k:1\).

Using section formula, the coordinates of point of intersection are given by,

\[
\left( \frac{k(-5) + 1}{k + 1}, \frac{k(4) + 4}{k + 1}, \frac{k(8) + 7}{k + 1} \right)
\]

On the \( yz \)-plane, the \( x \)-coordinate of any point is zero.

\[
\frac{3k - 2}{k + 1} = 0
\]

\[
\Rightarrow \frac{2}{3}
\]

\[
\Rightarrow k = \frac{2}{3}
\]

Thus the \( yz \)-plane divides the line segment formed by joining the given points in the ratio \(2:3\).

---

#418360

**Topic:** Section Formula

Using section formula show that the points \( A(2, -3, 4), B(-1, 2, 1) \) and \( C\left(0, \frac{1}{3}, 2\right) \) are collinear.

Solution

The given points are \( A(2, -3, 4), B(-1, 2, 1) \) and \( C\left(0, \frac{1}{3}, 2\right) \).

Let \( P \) be a point that divides \( AB \) in the ratio \( k:1 \).

Using section formula, the coordinates of \( P \) are given by,

\[
\left( \frac{k(-1) + 2}{k + 1}, \frac{k(2) - 3}{k + 1}, \frac{k(1) + 4}{k + 1} \right)
\]

Now, we will find the value of \( k \) at which point \( P \) coincides with point \( C \):

\[
\Rightarrow \frac{-k + 2}{k + 1} = 0 \quad \text{we get} \quad k = 2
\]

For \( k = 2 \), the coordinates of point \( P \) are \( \left(0, \frac{1}{3}, 2\right) \).

i.e., \( \left(0, \frac{1}{3}, 2\right) \) is a point that divides \( AB \) externally in the ratio \( 2:1 \) and is the same as point \( P \).

Hence, points \( A, B \) and \( C \) are collinear.

---

#418361

**Topic:** Section Formula

Find the coordinates of the points which trisect the line segment joining the points \( P(4, 2, -6) \) and \( Q(10, -16, 6) \).

Solution

Given coordinates of \( P \) as \( (4, 2, -6) \) and \( Q \) as \( (10, -16, 6) \).

Let \( R \) and \( S \) be the points of trisection of line segment \( PQ \).

Then \( R \) divides \( PQ \) in the ratio \( 1:2 \) and \( S \) is the mid-point of \( RQ \).

Let the coordinates of \( R \) be \((x, y, z)\).

By using section formula,

\[
x = \frac{10 + 4}{1 + 2}, \quad y = \frac{-16 + 2}{1 + 2}, \quad z = \frac{10 + 2}{1 + 2}
\]

\[
x = \frac{14}{3}, \quad y = -6, \quad z = \frac{12}{3}
\]

\[
x = 6, \quad y = -4, \quad z = -2
\]

So, the coordinates of \( R \) are \((6, -4, -2)\).

Now, since \( S \) is the mid-point of \( RQ \),
#418413

**Topic:** Distance Between Two Points

Find the lengths of the medians of the triangle with vertices A(0, 0, 6), B(0, 4, 0) and (6, 0, 0).

**Solution**

Let \( AD, BE \) and \( CF \) be the medians of the given \( \triangle ABC \).

Since \( AD \) is the median, \( D \) is the midpoint of \( BC \):

\[
\therefore \text{coordinates of point } D = \left( \frac{0 + 6}{2}, \frac{4 + 0}{2}, \frac{0 + 0}{2} \right) = (3, 2, 0)
\]

\[ AD = \sqrt{(3 - 0)^2 + (0 - 2)^2 + (6 - 0)^2} = \sqrt{9 + 4 + 36} \approx \sqrt{49} = 7 \]

Since \( BE \) is the median, \( E \) is the midpoint of \( AC \):

\[
\therefore \text{coordinates of point } E = \left( \frac{0 + 6}{2}, \frac{0 + 0}{2}, \frac{6 + 0}{2} \right) = (3, 0, 3)
\]

\[ BE = \sqrt{(3 - 0)^2 + (0 - 4)^2 + (3 - 0)^2} = \sqrt{9 + 16 + 9} \approx \sqrt{34} \]

Since \( CF \) is the median, \( F \) is the midpoint of \( AB \):

\[
\therefore \text{coordinates of point } F = \left( \frac{0 + 6}{2}, \frac{0 + 0}{2}, \frac{6 + 0}{2} \right) = (0, 2, 3)
\]

Length of \( CF = \sqrt{(6 - 0)^2 + (0 - 2)^2 + (0 - 3)^2} = \sqrt{36 + 4 + 9} \approx \sqrt{49} = 7 \)

Thus the lengths of the medians of \( \triangle ABC \) are 7, \( \sqrt{34} \) and 7 units.

#418434

**Topic:** Section Formula

If the origin is the centroid of the triangle \( PQR \) with vertices \( P(2a, 2, 6), Q(-4, 3b, -10) \) and \( R(8, 14, 2c) \) then find the values of \( a, b \) and \( c \).

**Solution**

The coordinates of the centroid of \( \triangle PQR \):

\[
\left( \frac{2a + 4}{3}, \frac{3b + 16}{3}, \frac{2c - 4}{3} \right)
\]

It is given that origin is the centroid of \( \triangle PQR \):

\[
(0, 0, 0) = \left( \frac{2a + 4}{3}, \frac{3b + 16}{3}, \frac{2c - 4}{3} \right)
\]

\[
\Rightarrow \frac{2a + 4}{3} = 0, \quad \frac{3b + 16}{3} = 0 \quad \text{and} \quad \frac{2c - 4}{3} = 0
\]

\[
\Rightarrow a = -2, \quad b = -\frac{16}{3} \quad \text{and} \quad c = 2
\]

#418443

**Topic:** Distance Between Two Points

Find the coordinates of a point on \( y \)-axis which are at a distance of \( 5\sqrt{2} \) from the point \( (3, -2, 5) \).

**Solution**

Let \( (0, b, 0) \) be the point on \( y \)-axis at a distance of \( 5\sqrt{2} \) from point \( (3, -2, 5) \).

We have given, \( AP = 5\sqrt{2} \)

\[
\Rightarrow AP^2 = 50
\]

\[
\Rightarrow (3 - 0)^2 + (-2 - b)^2 + (6 - 0)^2 = 50
\]

\[
\Rightarrow 9 + 4 + b^2 + 4b + 25 = 50
\]

\[
\Rightarrow b^2 + 4b - 12 = 0
\]

\[
\Rightarrow (b + 6)(b - 2) = 0
\]

\[
\Rightarrow b = -6, \quad 2
\]

Thus, the coordinates of the required points are \((0, 2, 0)\) and \((0, -6, 0)\).
A point \( P \) with \( x \)-coordinate 4 lies on the line segment joining the points \( P(2, -3, 4) \) and \( Q(8, 0, 10) \). Find the coordinates of the point \( P \).

**Solution**

The coordinates of points \( P \) and \( Q \) are given as \( P(2, -3, 4) \) and \( Q(8, 0, 10) \).

Let \( P \) divide line segment \( PQ \) in the ratio \( k:1 \).

Hence by section formula, the coordinates of point \( P \) are given by:

\[
\left( \frac{k(2) + 1(8)}{k + 1}, \frac{k(-3) + 1(0)}{k + 1}, \frac{k(4) + 1(10)}{k + 1} \right) = \left( \frac{8k + 2}{k + 1}, \frac{-3k + 4}{k + 1}, \frac{10k + 4}{k + 1} \right)
\]

It is given that the \( x \)-coordinate of point \( P \) is 4.

\[
\frac{8k + 2}{k + 1} = 4
\]

\[
8k + 2 = 4(k + 1)
\]

\[
8k + 2 = 4k + 4
\]

\[
k = 2
\]

Therefore, the coordinates of point \( P \) are \( \left( 4, \frac{-3}{2} + 1, \frac{10(2)}{2} + 4 \right) = (4, -2, 6) \).

---

If \( A \) and \( B \) be the points \((3, 4, 5)\) and \((-1, 3, -7)\) respectively. Find the equation of the set of points \( P \) such that \( PA^2 + PB^2 = k^2 \), where \( k \) is a constant.

**Solution**

Given coordinates of points \( A \) and \( B \) are \((3, 4, 5)\) and \((-1, 3, -7)\) respectively.

Let the coordinates of point \( P \) be \((x, y, z)\).

Given, \( PA^2 + PB^2 = k^2 \)

\[
(\sqrt{(x - 3)^2 + (y - 4)^2 + (z - 5)^2} + \sqrt{(x + 1)^2 + (y - 3)^2 + (z + 7)^2})^2 = k^2
\]

\[
(x^2 - 6x + 9 + y^2 - 8y + 16 + z^2 - 10z + 25) + (x^2 + 2x + 1 + y^2 - 6y + 9 + z^2 + 14z + 49) = k^2
\]

\[
2x^2 + 2y^2 + 2z^2 + 2x + 2y + 2z + 109 = k^2
\]

\[
2x^2 + 2y^2 + 2z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}
\]

which is the required equation.

---

Write the equations for the \( x \)- and \( y \)-axes.

**Solution**

The \( y \)-coordinate of every point on \( x \)-axis is 0.

Therefore, the equation of the \( x \)-axis is \( y = 0 \).

And the \( x \)-coordinate of every point on \( y \)-axis is 0.

Therefore, the equation of the \( y \)-axis is \( x = 0 \).

---

In what ratio, the line joining \((-1, 7)\) and \((3, 7)\) is divided by the line \( y + x = 4 \)?

**Solution**
The equation of the line joining the points \((-1, 1)\) and \((5, 7)\) is given by,
\[
\begin{align*}
y - 1 &= \frac{7 - 1}{5 - (-1)}(x + 1) \\
y - 1 &= \frac{6}{6}(x + 1) \\
x - y + 2 &= 0 \quad \text{...(1)}
\end{align*}
\]
and the equation of the given line is
\[
x + y - 4 = 0 \quad \text{...(2)}
\]
Thus point of intersection of lines (1) and (2) is given by,
\[
x = 1 \text{ and } y = 3
\]
Let point \((1, 3)\) divide the line segment joining \((-1, 1)\) and \((5, 7)\) in the ration \(1 : k\).
Accordingly, by section formula,
\[
\begin{align*}
(1, 3) &= \left[\frac{k(-1) + 1(5)}{1 + k}, \frac{k(1) + 1(7)}{1 + k}\right] \\
\Rightarrow (1, 3) &= \left[\frac{-k + 5}{1 + k}, \frac{k + 7}{1 + k}\right] \\
\Rightarrow \frac{-k + 5}{1 + k} &= 1, \frac{k + 7}{1 + k} = 3 \\
\Rightarrow -k + 5 &= 1, k + 7 = 3k + 3 \\
\Rightarrow -k + 5 &= 1, k &= 2
\end{align*}
\]
Thus, the line joining the points \((-1, 1)\) and \((5, 7)\) is divided by line \(x + y = 4\) in the ratio \(1 : 2\).