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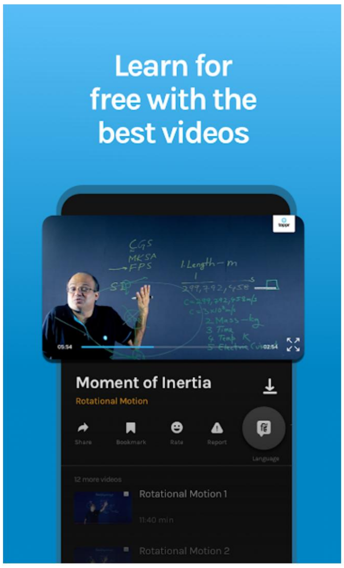

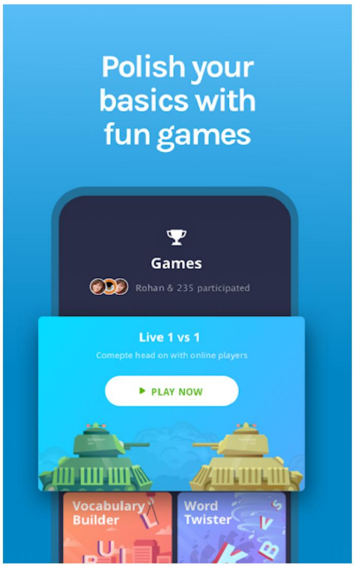

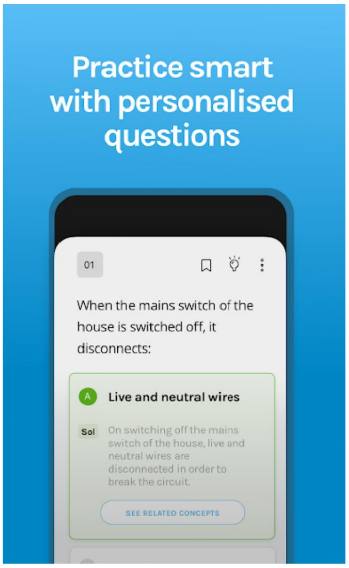

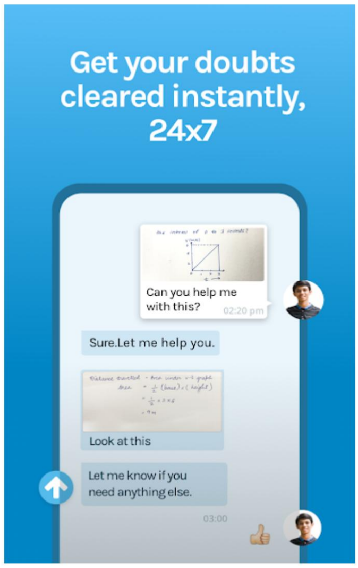

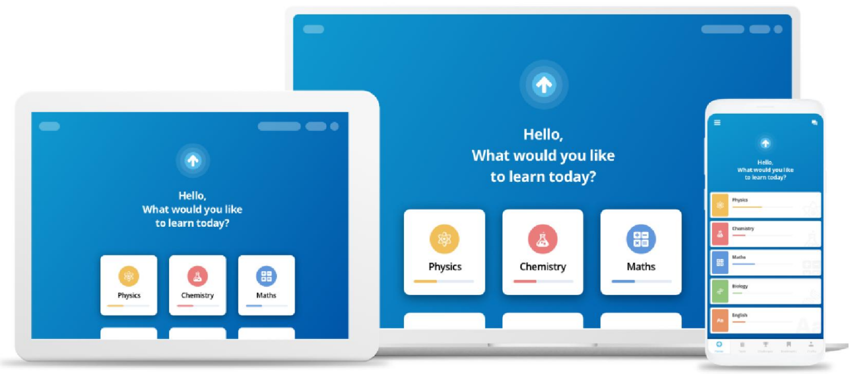

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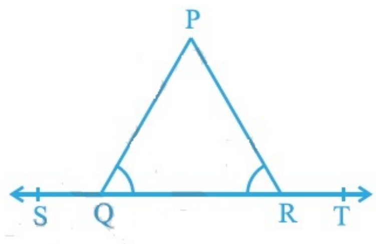
#463644
Topic: Properties of Triangles



In the figure, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.

Solution

As Given, $\angle PQR = \angle PRQ$

To prove: $\angle PQS = \angle PRT$

According to the question,

$\angle PQR + \angle PQS = 180°$ | Linear Pair

$\Rightarrow \angle PQS = 180° - \angle PQR$--- (i)

Also $\angle PRQ + \angle PRT = 180°$ | Linear Pair

$\Rightarrow PRT = 180° - \angle PRQ$

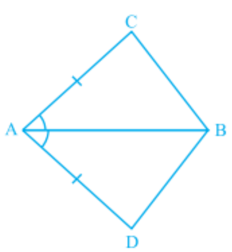
$\Rightarrow \angle PRQ = 180° - \angle PQR - - - (ii)(\angle PQR = \angle PRQ)$

From (i) and (ii),

$\angle PQS = \angle PRT = 180° - \angle PQR$

$\therefore \angle PQS = \angle PRT$

#463822
Topic: Congruent Triangles



In quadrilateral $ACBD$, $AC = AD$ and AB bisects $\angle A$. Show that $\triangle ABC \equiv \triangle ABD$. What can you say about BC and BD?

Solution

In $\triangle ABC$ and $\triangle ABD$,

$AC = AD$ (Given)

$\angle CAB = \angle DAB$ (AB bisects $\angle A$)

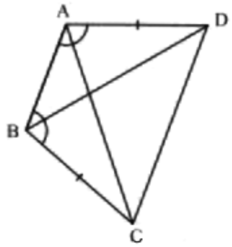
$AB = AB$ (Common)

$\therefore \triangle ABC \cong \triangle ABD$ (By SAS congruence rule)

$\therefore BC = BD$ (By CPCT)

$\therefore$, BC and BD are of equal lengths.

#463824
Topic: Congruent Triangles



$ABCD$ is a quadrilateral in which $AD = BC$ and $\angle DAB = \angle CBA$ (see Fig.). Prove that

(i) $\triangle ABD \cong \triangle BAC$

(ii) $BD = AC$

(iii) $\angle ABD = \angle BAC$

Solution

In $\triangle ABD$ and $\triangle BAC$,

$AD = BC$ (Given)

$\angle DAB = \angle CBA$ (Given)

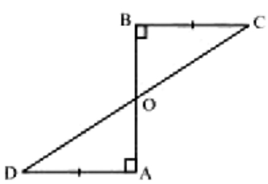
$AB = BA$ (Common)

$\therefore \triangle ABD \cong \triangle BAC$ (By SAS congruence rule)

$\therefore BD = AC$ (By CPCT)

And, $\angle ABD = \angle BAC$ (By CPCT)

#463825
Topic: Congruent Triangles



AD and BC are equal perpendiculars to a line segment AB (see Fig). Show that CD bisects AB

Solution

In $\triangle BOC$ and $\triangle AOD$,

$\angle BOC = \angle AOD$ (Vertically opposite angles)

$\angle CBO = \angle DAO$ (Each 90)

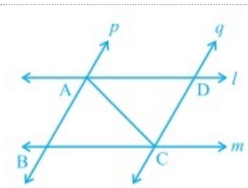
$BC = AD$(Given)

$\therefore \triangle BOC \cong \triangle AOD$ (AAS congruence rule)

$\therefore BO = AO$ (By CPCT)

CD bisects AB.

#463826
Topic: Congruent Triangles



l and m are two parallel lines intersected by another pair of parallel lines p and q. Show that $\triangle ABC \cong \triangle CDA$

Solution

In $\triangle ABC$ and $\triangle CDA$

$\angle BAC = \angle DCA$ (Alternate interior angles, as $p \parallel q$)

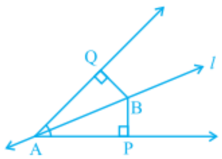
$AC = CA$ (Common)

$\angle BCA = \angle DAC$ (Alternate interior angles, as $l \parallel m$)

$\therefore \triangle ABC \cong \triangle CDA$ (By ASA congruence rule)

#463827

Topic: Congruent Triangles



Line l is the bisector of an angle $\angle A$ and B is any point on l. BP and BQ are perpendiculars from B to the arms of $\angle A$. Show that:

(i) $\triangle ABP \cong \triangle AQB$

(ii) $BP = BQ$ or B is equidistant from the arms of $\angle A$

Solution

In $\triangle APB$ and $\triangle AQB$

$\angle APB = \angle AQB$ (Each 90^o)

$\angle PAB = \angle QAB$ (l is the angle bisector of $\angle A$)

$AB = AB$ (Common)

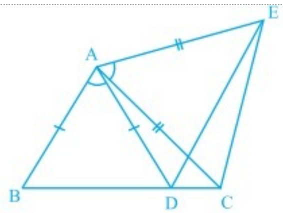
$\therefore \triangle APB \cong \triangle AQB$ (By AAS congruence rule)

$\therefore BP = BQ$ (By CPCT)

It can be said that B is equidistant from the arms of $\angle A$.

#463828

Topic: Congruent Triangles



In Fig, $AC = AE$, $AB = AD$ and $\angle BAD = \angle EAC$. Show that $BC = DE$

Solution

It is given that $\angle BAD = \angle EAC$

$\angle BAD + \angle DAC = \angle EAC + \angle DAC$

$\angle BAC = \angle DAE$

In $\triangle BAC$ and $\triangle DAE$

$AB = AD$ (Given)

$\angle BAC = \angle DAE$ (Proved above)

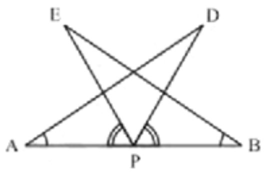
$AC = AE$ (Given)

$\therefore \triangle BAC \cong \triangle DAE$ (By SAS congruence rule)

$\therefore BC = DE$ (By CPCT)

#463829

Topic: Congruent Triangles



AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$. Show that

(i) $\triangle DAP \cong \triangle EBP$

(ii) $AD = DE$

Solution

i)

It is given that $\angle EPA = \angle DPB$

Now,

$\angle EPA + \angle DPE = \angle DPB + \angle DPE$

Therefore,

$\angle DPA = \angle EPB$

In ΔEBP and ΔDAP,

$\angle EBP = \angle DAP$ (given)

$BP = AP$ (P is midpoint of AB)

$\angle EPB = \angle DPA$ [proved above]

By ASA criterion of congruence,

$\Delta EBP \cong \Delta DAP$

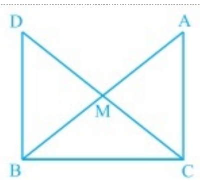
ii)

Since $\Delta EBP \cong \Delta DAP$

$AD = BE$ (using CPCT)

#463831

Topic: Congruent Triangles



In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that $DM = CM$. Point D is joined to point B (see Fig.).

Show that:

(i) $\triangle AMC \cong \triangle BMD$

(ii) $\angle DBC$ is a right angle.

(iii) $\triangle DBC \cong \triangle ACB$

(iv) $CM = \dfrac{1}{2}AB$

Solution

(i)

In ΔAMC and ΔBMD,

$\because$ M is the mid- point of AB

$AM = BM$

$\angle AMC = \angle BMD$ (Vertically opp. angles)

$CM = MD$ (Given)

$\therefore \Delta AMC \cong \Delta BMD$ (SAS test of congruence)

(ii)

$\Delta AMC \cong \Delta BMD$ ...(1)

$\therefore BD = CA$ (CPCT)

$\angle BDM = \angle ACM$ (CPCT)

i.e.

$\angle BDC = \angle ACD$

$\therefore BD \parallel CA$. (Alternate angle theorem)

$\therefore \angle CBD + \angle BCA = 180^\circ$

(Sum of interior angles between parallel lines is 180^o)

$\Rightarrow \angle CBD + 90^\circ = 180^\circ$

$\Rightarrow \angle CBD = 180^\circ - 90^\circ$

$\therefore \angle DBC = 90^\circ$

(iii)

In ΔDBC and ΔACB, we have

$BD = CA$ ...from (1)

$\angle DBC = \angle ACB$ (Each 90^o)

$BC = BC$ (Common side)

$\Delta DBC \cong \Delta ACB$ (SAS test of congruence)

(iv)

$\therefore CD = AB$

$\Rightarrow CM = DM = \dfrac{1}{2}CD$

$\because$ M is the midpoint of CD

$\therefore CM = \dfrac{1}{2}CD$

$\therefore CM = \dfrac{1}{2}AB$

#463832

Topic: Congruent Triangles

In an isosceles triangle ABC, with $AB = AC$, the bisectors of $\angle B$ and $\angle C$ intersect each other at O. Join A to O. Show that :

(i) $OB = OC$ (ii) AO bisects $\angle A$

Solution

(i)

In ΔABC, we have

$AB = AC$

$\therefore \angle ACB = \angle ABC$ (Isosceles triangle theorem)

$\therefore \frac{1}{2}\angle ACB = \frac{1}{2}\angle ABC$ ..(1)

$\therefore \angle OCB = \angle OBC$ and $\angle ACO = \angle ABO$

[OC and OB are bisectors of $\angle C$ and $\angle B$ respectively]

$\therefore OC = OB$ (Converse of isosceles triangle theorem) ... (2)

(ii)

In ΔABO and ΔACO

$AB = AC$ (Given)

$\angle ABO = \angle ACO$ ...from (1)

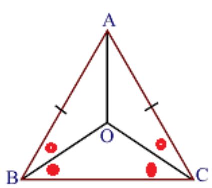
$OB = OC$ ...from (2)

$\therefore \Delta ABO \cong \Delta ACO$ (SAS test of congruence)

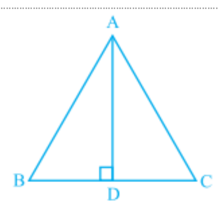
$\therefore \angle OAB = \angle OAC$ (CPCT)

So,

AO bisects $\angle A$



#463833
Topic: Congruent Triangles



In $\triangle ABC$, AD is the perpendicular bisector of BC. Show that $\triangle ABC$ is an isosceles triangle in which $AB = AC$.

Solution

$\because$ AD is the perpendicular bisector of BC

$DB = DC$

In ΔABD and ΔACD,

$AD = AD$ (Common side)

and

$DB = DC$

$\angle ADB = \angle ADC$

(Both are 90^o since $AD \perp BC$)

By SAS criterion of congruence,

$\Delta ABD \cong \Delta ACD$

So,

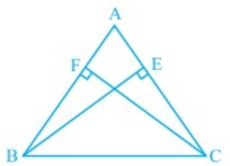
$AB = AC$ (by CPCT)

Therefore,

ΔABC is isosceles triangle.

#463834

Topic: Congruent Triangles



ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see Fig.). Show that these altitudes are equal.

Solution

In ΔABE and ΔACF,

$AB = AC$

$\because$ ΔABC is an isosceles triangle

$\angle BAE = \angle CAF$

Common since both equal to $\angle A$

$\angle AEB = \angle AFC$ (Both equal to 90^o)

By AAS criterion of congruence,

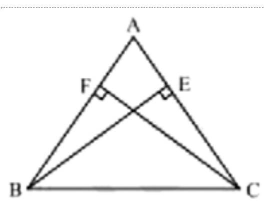
$\Delta ABE \cong \Delta ACF$

So,

$BE = CF$ (by CPCT)

#463835

Topic: Congruent Triangles



ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see Fig.). Show that

(i) $\triangle ABE \cong \triangle ACF$

(ii) $AB = AC$, i.e., ABC is an isosceles triangle

Solution

In ΔABE and ΔACF,

$\angle BAE = \angle CAF$ (Common angle)

$\angle AEB = \angle AFC$

$\because BE \perp AC$ and $CF \perp AB$

and

$BE = CF$ (Given that altitudes are equal)

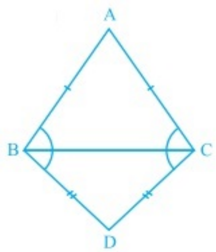
By AAS criterion of congruence,

$\Delta ABE \cong \Delta ACF$

Hence,

$AB = AC$ (by CPCT)

#463837
Topic: Properties of Triangles



ABC and DBC are two isosceles triangles on the same base BC (see Fig.). Show that $\angle ABD = \angle ACD$.

Solution

Since ΔABC is isoceles,

So, $AB = AC$

$\Rightarrow \angle ABC = \angle ACB$ ...(1)

(Angles opposite to equal sides are equal)

Since, ΔBCD is an isosceles triangle,

$BD = CD$

$\Rightarrow \angle DBC = \angle DCB$ ...(2)

(Angles opposite to equal sides are equal)

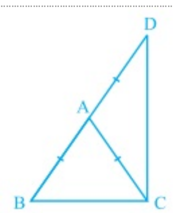
On adding 1 and 2, we get

$\angle ABC + \angle DBC = \angle ACB + \angle DCB$

This can be written as

$\angle ABD = \angle ACD$.

#463839
Topic: Properties of Triangles



$\triangle ABC$ is an isosceles triangle in which $AB = AC$. Sides BA is produced to D such that $AD = AB$. Show that $\angle BCD$ is a right angle.

Solution

In ΔADC,

$AD = AC$

($\because AB = AC$ and it is given that $AD = AB$)

So, $AD = AC$

$\angle ACD = \angle ADC$ ...(1)

(Angles opp. to equal sides are equal)

ΔABC is an isosceles triangle,

and

$AB = AC$

So, $\angle ACB = \angle ABC$ ...(2)

(Angles opp. to equal sides are equal)

On adding (1) and (2),

$\angle ACB + \angle ACD = \angle ABC + \angle ADC$

$\Rightarrow \angle BCD = \angle ABC + \angle BDC$

($\because \angle ADC = \angle BDC$ are same)

Adding $\angle BCD$ on both sides,

$\angle BCD + \angle BCD = \angle ABC + \angle BDC + \angle BCD$

Now,

$\angle ABC + \angle BDC + \angle BCD = 180^\circ$

(Angle sum property of a triangle)

$2\angle BCD = 180^\circ$

$\angle BCD = 90^\circ$

$\angle BCD$ is a right angle.

#463841

Topic: Properties of Triangles

ABC is a right angled triangle in which $\angle A = 90^\circ$ and $AB = AC$. Find $\angle B$ and $\angle C$.

Solution

Since $AB = AC$,

So, $\triangle ABC$ is isosceles.

$\angle B = \angle C$ ...(angles opp. to equal sides are equal)

$\angle A + \angle B + \angle C = 180^\circ$ ...(angle - sum property of a triangle)

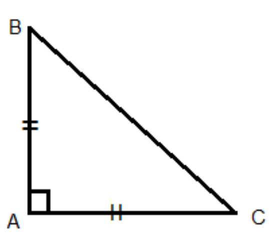
Substituting $\angle B = \angle C,\ \angle A = 90^o$

$90^\circ + 2\angle B = 180^\circ$

$2\angle B = 180^\circ - 90^\circ = 90^\circ$

$\Rightarrow \angle B = 45^\circ$

So, $\angle C = \angle B = 45^o$.



#463842

Topic: Properties of Triangles

Show that the angles of an equilateral triangle are 60° each.

Solution

For an equilateral triangle, all sides are equal.

Assuming an equilateral ΔABC,

Then,

$AB = AC = BC$.

$\Rightarrow \angle A = \angle B = \angle C$

(Angles opp. to equal sides are equal)

For a triangle, by angle sum property,

$\angle A + \angle B + \angle C = 180^\circ$

Substituting

$\Rightarrow \angle A = \angle B = \angle C$

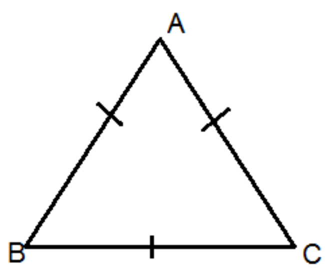
$\therefore \angle A + \angle A + \angle A = 180^\circ$

$\Rightarrow 3\angle A = 180^\circ$

$\Rightarrow \angle A = 60^\circ$

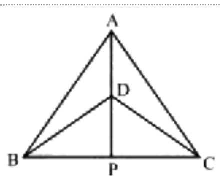
$\therefore \angle A = \angle B = \angle C = 60^\circ$

So, each angle of an equilateral triangle is 60^o.



#463854

Topic: Congruent Triangles



$\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC. If AD is extended to intersect BC at P, show that

(i) $\triangle ABD \cong \triangle ACD$

(ii) $\triangle ABP \cong \triangle ACP$

(iii) AP bisects $\angle A$ as well as $\triangle D$

(iv) AP is the perpendicular bisector of BC

Solution

(i) In $\triangle ABD$ and $\triangle ACD$,

$AB = AC$ ....(since $\triangle ABC$ is isosceles)

$AD = AD$ ....(common side)

$BD = DC$ ....(since $\triangle BDC$ is isosceles)

$\Delta ABD \cong \Delta ACD$ .....SSS test of congruence,

$\therefore \angle BAD = \angle CAD$ i.e. $\angle BAP = \angle PAC$ .....c.a.c.t.

(ii) In $\triangle ABP$ and $\triangle ACP$,

$AB = AC$ ...(since $\triangle ABC$ is isosceles)

$AP = AP$ ...(common side)

$\angle BAP = \angle PAC$ ....from (i)

$\triangle ABP \cong \triangle ACP$ .... SAS test of congruence

$\therefore BP = PC$ ...c.s.c.t.

$\angle APB = \angle APC$ ....c.a.c.t.

(iii) Since $\triangle ABD \cong \triangle ACD$

$\angle BAD = \angle CAD$ ....from (i)

So, AD bisects $\angle A$

i.e. AP bisects $\angle A$

In $\triangle BDP$ and $\triangle CDP$,

$DP = DP$ ...common side

$BP = PC$ ...from (ii)

$BD = CD$ ...(since $\triangle BDC$ is isosceles)

$\triangle BDP \cong \triangle CDP$ ....SSS test of congruence

$\therefore \angle BDP = \angle CDP$ ....c.a.c.t.

$\therefore$ DP bisects $\angle D$

So, AP bisects $\angle D$ ....(iv)

From (iii) and (iv),

AP bisects $\angle A$ as well as $\angle D$.

(iv) We know that

$\angle APB + \angle APC = 180^\circ$ ....(angles in linear pair)

Also, $\angle APB = \angle APC$ ...from (ii)

$$\therefore \angle APB = \angle APC = \frac{180^\circ}{2} = 90^\circ$$

$BP = PC$ and $\angle APB = \angle APC = 90^\circ$

Hence, AP is perpendicular bisector of BC.

#463855

Topic: Congruent Triangles

AD is an altitude of an isosceles triangle ABC in which $AB = AC$. Show that

(i) AD bisects BC (ii) AD bisects $\angle A$

Solution

In ΔABC,

AD is the altitude drawn from vertex A to side BC

$\therefore \angle D = 90^o$

and $AB = AC$ (Given)

In ΔADB and ΔADC,

Hypotenuse $AB =$ Hypotenuse AC (Given)

Side $AD =$ Side AD (Common Side)

$\angle ADC = \angle ADB$

$\Delta ADB \cong \Delta ADC$

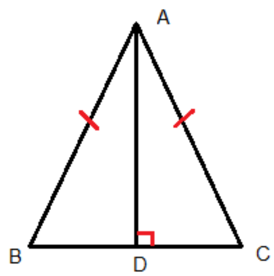
$BD = DC$ (CPCT)

$\therefore D$ is the midpoint of BC,

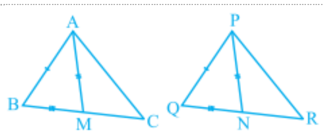
i.e. AD bisects BC.

$\angle BAD = \angle DAC$ (CPCT)

AD is bisector of $\angle A$.



#463856
Topic: Congruent Triangles



Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of $\triangle PQR$ (see Fig). Show that:

(i) $\triangle ABM \cong \triangle PQN$

(ii) $\triangle ABC \cong \triangle PQR$

Solution

In ΔABC and ΔPQR

Given:

$AB = PQ$

$AM = PN$

$BC = QR$

$\therefore \frac{1}{2}BC = \frac{1}{2}QR$

$\therefore BM = QN$ ...(1)

In ΔABM and ΔPQN,

Given:

$AB = PQ$

$BM = QN$ ... from (1)

$AM = PN$

$\therefore \Delta ABM \cong \Delta PQN$ (By SSS test of congruence)

$\therefore \angle B = \angle Q$ (CPCT) ...(2)

In ΔABC and ΔPQR,

$AB = PQ$ (Given)

$BC = QR$ (Given)

$\angle B = \angle Q$ ... from (2)

$\therefore \Delta ABC \cong \Delta PQR$ (By SAS test of congruence)

#463858

Topic: Congruent Triangles

BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Solution

In ΔBCF and ΔCBE,

$\angle BFC = \angle CEB$ (Each 90^o)

Hyp. $BC =$ Hyp. BC (Common Side)

Side $FC =$ Side EB (Given)

$\therefore$ By R.H.S. criterion of congruence, we have

$\Delta BCF \cong \Delta CBE$

$\therefore \angle FBC = \angle ECB$ (CPCT)

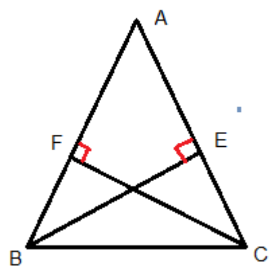
In ΔABC,

$\angle ABC = \angle ACB$

[$\because \angle FBC = \angle ECB$]

$\therefore AB = AC$ (Converse of isosceles triangle theorem)

$\therefore \Delta ABC$ is an isosceles triangle.



#463859
Topic: Congruent Triangles

ABC is an isosceles triangle with $AB = AC$. Draw $AP \perp BC$ to show that $\angle B = \angle C$

Solution

In ΔABP and ΔACP,

$\angle APB = \angle APC$ (Both equal to 90^0)

$AB = AC$

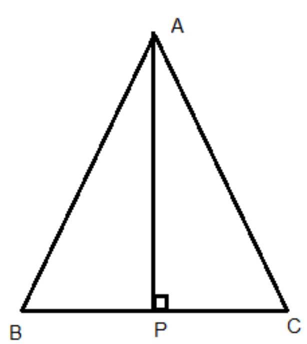
$\because \Delta ABC$ is an isosceles triangle.

$AP = AP$ (Common Side)

By R.H.S. criterion of congruence,

$\triangle ABP \cong \triangle ACP$

$\Rightarrow \angle B = \angle C$ (CPCT)



In ΔABP and ΔACP,

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$\because \Delta ABC$ is an isosceles triangle.

$AP = AP$ (Common Side)

By R.H.S. criterion of congruence,

$\triangle ABP \cong \triangle ACP$

$\Rightarrow \angle B = \angle C$ (CPCT)

#463861

Topic: Properties of Triangles

Show that in a right angled triangle, the hypotenuse is the longest side.

Solution

Consider ΔPQR which is right angled at Q.

By angle sum property of a triangle,

$\angle PQR + \angle PRQ + \angle QPR = 180^\circ$

$\Rightarrow 90^\circ + \angle R + \angle P = 180^\circ$

$\Rightarrow \angle R + \angle P = 90^\circ$

$\Rightarrow \angle R$ and $\angle P$ are acute angles

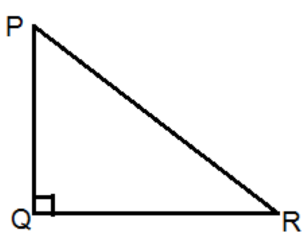
$\Rightarrow \angle R < 90^\circ$ and $\angle P < 90^\circ$

$\Rightarrow \angle R < \angle Q$ and $\angle P < \angle Q$

$\Rightarrow PR > PQ$ and $PR > QR$

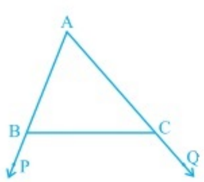
(Side opposite to greater angle is greater)

So, the hypotenuse is the longest side in a triangle.



#463863

Topic: Properties of Triangles



In the figure, sides AB and AC of $\triangle ABC$ are extended to points P and Q respectively. Also, $\angle PBC < \angle QCB$. Show that $AC > AB$.

Solution

$\angle PBC < \angle QCB$ (Given)

Multiply the equation by -1.

$\Rightarrow \ -\angle PBC > \ -\angle QCB$

Adding 180° on both sides, we get

$\therefore 180^\circ - \angle PBC > 180^\circ - \angle QCB$

Angles on a straight line add to 180^o

Sum of angles $\angle PBC$ and $\angle ABC$ is 180^o.

Sum of angles $\angle QCB$ and $\angle ACB$ is 180^o.

So,

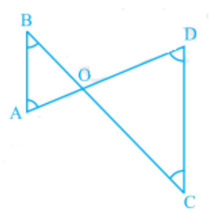
$\Rightarrow \angle ABC > \angle ACB$

$\Rightarrow AC > AB$

(Side opposite to greater angle is greater)

#463864

Topic: Properties of Triangles



In fig, $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$.

Solution

It is given that

$\angle B < \angle A$ and $\angle C < \angle D$

We know that side opposite to larger angle is larger

$OD < OC$ .... (i)

$AO < BO$ .... (ii)

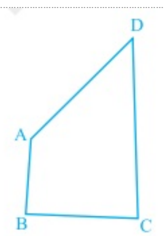
Adding eq(i) and eqn (ii), we get

$AO + OD < BO + OC$

$\Rightarrow AD < BC$

#463865

Topic: Properties of Triangles



AB and CD are respectively the smallest and longest sides of a quadrilateral $ABCD$. Show that $\angle A > \angle C$ and $\angle B > \angle D$.

Solution

In ΔABC,

$BC > AB$ ...(AB is the smallest side, given)

$\angle BAC > \angle BCA$ ...(i)

Similarly, in ΔACD,

$CD > AD$ ......(CD is the greatest side, given)

$\therefore \angle CAD > \angle ACD$ ...(ii)

Adding (i) and (ii), we have

$\angle BAC + \angle CAD > \angle BCA + \angle ACD$

$\Rightarrow \angle A > \angle C$

Now, in ΔABD,

$AD > AB$ ...Gievn

$\therefore \angle ABD > \angle ADB$ ...(iii)

Similarly, in ΔBCD,

$CD > BC$

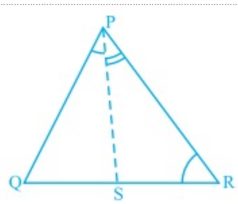
$\angle DBC > \angle BDC$ ...(iv)

Adding (iii) and (iv), we have

$\angle ABC > \angle ADC$

$\Rightarrow \angle B > \angle D$

#463867
Topic: Properties of Triangles



In fig, $PR > PQ$ and PS bisects $\angle QPR$. Prove that $\angle PSR > \angle PSQ$.

Solution

In ΔPQR,

$PR > PQ$ (Given)

$\Rightarrow \angle PQR > \angle PRQ$ ...(1)

(Angle opposite to side of greater length is greater

PS is the bisector of $\angle P$, so $\angle x = \angle y$

Adding $\angle x$ in (1)

$\Rightarrow \angle PQR + \angle x > \angle PRQ + \angle x$

$\Rightarrow \angle PQR + \angle x > \angle PRQ + \angle y$ .... (2)

In ΔPQS,

$\angle PQS + \angle x + \angle PSQ = 180^\circ$

(Angle sum property of triangle)

$\therefore \angle PQS + \angle x = 180^\circ - \angle PSQ$ ..... (3)

In ΔPSR ,

$\angle PRS + \angle y + \angle PSR = 180^\circ$

(Angle sum property of triangle)

$\angle PRS + \angle y = 180^\circ - \angle PSR$

Using equation (1), (2), (3) we get

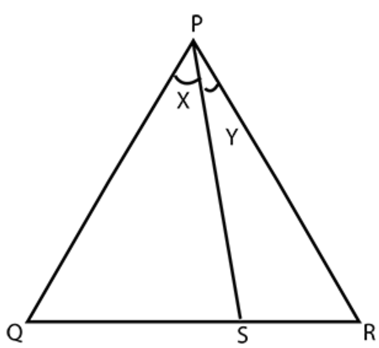
$180^\circ - \angle PSQ > 180^\circ - \angle PSR$ ...(4)

$\Rightarrow -\angle PSQ > -\angle PSR$

$\Rightarrow \angle PSQ < \angle PSR$

So,

$\angle PSR > \angle PSQ$



#463869

Topic: Theorems of Triangles

ABC is a triangle. Locate a point in the interior of $\triangle ABC$ which is equidistant from all the vertices of $\triangle ABC$.

Solution

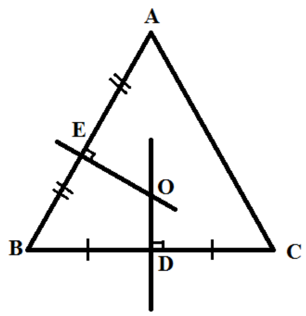
Let OD and OE be the perpendicular bisectors of sides BC and AB of $\triangle ABC$ respectively.

$\therefore$ By perpendicular bisector theorem,

O is equidistant from the end points of seg BC i.e. points B and C.

Similarly, point O is equidistant from end points of seg AC i.e points C and A.

Hence, the point of intersection O of the perpendicular bisectors of sides AB and BC is equidistant from vertices A, B, C of $\triangle ABC$.



#463870

Topic: Theorems of Triangles

In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.

Solution

Let BQ and CP be the bisectors of $\angle ABC$ and $\angle ACB$ respectively, intersecting in the interior of $\triangle ABC$ at R.

Let BQ intersect side AC in Q and CP intersect side AB in P.

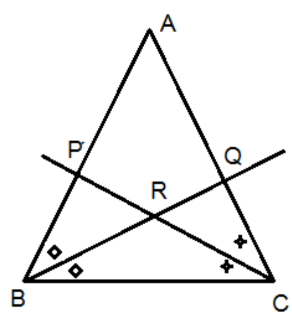
$\therefore$ By angle bisector theorem,

Since, R lies on BQ, point R is equidistant from AB and BC.

Similarly, R lies on CP and is equidistant from AC and BC.

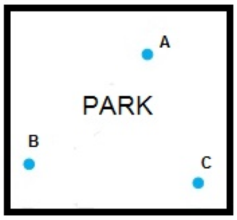
So, O is equidistant from BC and AC.

Therefore, point O is equidistant from all three sides AB, BC and CA of $\triangle ABC$.



#463872

Topic: Theorems of Triangles



In a huge park, people are concentrated at three points (see Fig):

A : where there are different slides and swings

B : near which a man-made lake is situated,

C : which is near to a large parking and exit

Where should an icecream parlour be set up so that maximum number of persons can approach it?

Solution

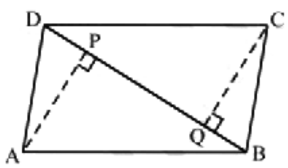
To set up a parlor, we should chose a place that is equidistant from A, B and C.

This point can be located by obtaining point of intersection of perpendicular bisector.

So, D is the required point which is equidistant from A, B and C.

#463884

Topic: Congruent Triangles



$ABCD$ is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD. Show that

(i) $\triangle APB \cong \triangle CQD$

(ii) $AP = CQ$

Solution

(i) In $\triangle APB$ and $\triangle CQD$,

$\angle APB = \angle CQD = 90^\circ$ ....given

$\angle ABP = \angle QDC$ ....(alternate interior angles of parallelogram $ABCD$ and $DC \parallel AB$)

$AB = CD$ ....Opposite sides of a || gm

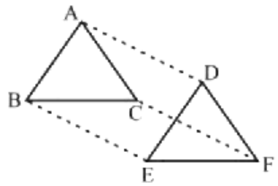
$\therefore \triangle APB \cong \triangle CQD$ ....SAA test of congruence

(ii) $\triangle APB \cong \triangle CQD$ ...from (i)

$\therefore AP = CQ$ ....c.s.c.t

#463885

Topic: Congruent Triangles



In $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to vertices D, E and F respectively. Show that

(i) Quadrilateral $ABED$ is a parallelogram

(ii) Quadrilateral $BEFC$ is a parallelogram

(ii) $AD \parallel CE$ and $AD = CF$

(iv) Quadrilateral $ACFD$ is a parallelogram

(v) $AC = DF$

(vi) $\triangle ABC \cong \triangle DEF$

Solution

(i) Consider the quadrilateral $ABED$

We have , $AB = DE$ and $AB \parallel DE$

One pair of opposite sides are equal and parallel. Therefore

$ABED$ is a parallelogram.

(ii) In quadrilateral $BEFC$, we have

$BC = EF$ and $BC \parallel EF$. One pair of opposite sides are equal and parallel.therefore ,$BEFC$ is a parallelogram.

(iii) $AD = BE$ and $AD \parallel BE$ | As $ABED$ is a llgm ... (1)

and $CF = BE$ and $CF \parallel BE$ | As $BEFC$ is a llgm ... (2)

From (1) and (2) , it can be inferred

$AD = CF$ and $AD \parallel CF$

(iv) $AD = CF$ and $AD \parallel CF$

$\Rightarrow$ One pair of opposite sides are equal and parallel.

$\Rightarrow$ $ACFD$ is a parallelogram.

(v) Since $ACFD$ is parallelogram.

$AC = DF$ | As Opposite sides of all gm $ACFD$

(vi) In triangles ABC and DEF, we have

$AB = DE$ | (opposite sides of $ABED$

$BC = EF$ | (Opposite sides of $BEFC$

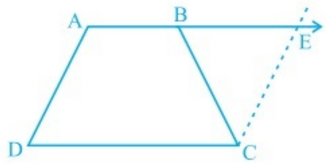
and $CA = FD$ | Opposite. sides of $ACFD$

Using SSS criterion of congruence,

$\triangle ABC \cong \triangle DEF$

#463886

Topic: Congruent Triangles



$ABCD$ is a trapezium in which $AB \parallel CD$ and $AD = BC$. Show that

(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\triangle ABC \cong \triangle BAD$

(iv) diagonal $AC =$ diagonal BD

Solution

Given :

$ABCD$ is a trapezium in which $AB \parallel CD$ and $AD = BC$

To prove :

(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\triangle ABC \cong \triangle BAD$

(iv) Diagonal $AC =$ diagonal BD.

Proof:

$AD \parallel CE$

AE is transversal and AE cuts them at A and E respectively.

Therefore, $\angle A + \angle E = 180^o$ ...(1)

Since $AB \parallel CD$ and $AD \parallel CE$.

$AECD$ is a parallelogram .

Therefore,

$\Rightarrow AD = CE$

$\Rightarrow BC = CE$ (Since $AD = BC$ (given))

Thus, in $\triangle BCE$

$BC = CE$ (By Angle sum property)

$\angle CEB = \angle CBE$

$180^\circ - \angle B = \angle E$

$180^\circ - \angle E = \angle B$

$\therefore \angle A = \angle B$

(ii) $\angle BAD = \angle ABD$

$180^\circ - \angle BAD = 180^\circ - \angle ABD$

$\angle ADB = \angle BCD$

$\angle D = \angle C$ i.e. $\angle C = \angle D$

(iii) In $\triangle ABC$ and $\triangle BAD$, we have

$BC = AD$ (Given)

$AB = BA$ (Common)

$\angle A = \angle B$ proved

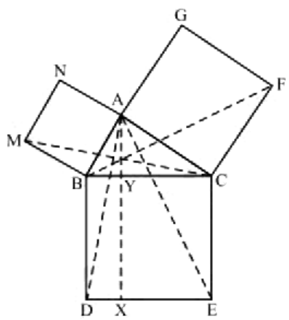
Using SAS criterion of congruence

$\triangle ABC \cong \triangle BAD$

(iv) Therefore, $AC = BD$ (CPCT)

#463951

Topic: Congruent Triangles



In fig, ABC is a right triangle right angled at A. $BCED$, $ACFG$ and $ABMN$ are square on the sides BC, CA and AB respectively. Line segment $AX \perp DE$ meets BC at Y. Show that:

(i) $\triangle MBC \cong \triangle ABD$

(ii) $ar(BYXD) = 2ar(MBC)$

(iii) $ar(BYXD) = ar(ABMN)$

(iv) $\triangle FCB \cong \triangle ACE$

(v) $ar(CYXE) = 2ar(FCB)$

(vi) $ar(CYXE) = ar(ACFG)$

(vii) $ar(BCED) = ar(ABMN) + ar(ACFG)$

Result (vii) is the famous Theorem of Pythagoras. You shall learn a simpler

proof of this theorem in Class X.

Solution

(i)

In $\Delta s\ MBC$ and ABD, we have

$BC = BD$

[Sides of the square BCED]

$MB = AB$

[Sides of the square ABMN]

$\angle MBC = \angle ABD$

[Since Each $= 90^o + \angle ABC$]

Therefore by SAS criterion of congruence, we have

$\Delta MBC \cong \delta ABD$

(ii)

ΔABD and square $BYXD$ have the same base BD and are between the same parallels BD and AX.

Therefore $ar(ABD) = \frac{1}{2} ar(BYXD)$

But $\Delta MBC \cong \Delta ABD$ [Proved in part (i)]

$\Rightarrow ar(MBC) = ar(ABD)$

Therefore $ar(MBC) = ar(ABD) = \frac{1}{2} ar(BYXD)$

$\Rightarrow ar(BYXD) = 2ar(MBC)$.

(iii)

Square $ABMN$ and ΔMBC have the same base MB and are between same parallels MB and NAC.

Therefore $ar(MBC) = \frac{1}{2} ar(ABMN)$

$\Rightarrow ar(ABMN) = 2ar(MBC)$

$= ar(BYXD)$ [Using part (ii)]

(iv)

In ΔsACE and BCF, we have

$CE = BC$ [Sides of the square BCED]

$AC = CF$ [Sides of the square ACFG]

and $\angle ACE = \angle BCF$ [Since Each $= 90^o + \angle BCA$]

Therefore by SAS criterion of congruence,

$\Delta ACE \cong \Delta BCF$

(v)

ΔACE and square $CYXE$ have the same base CE and are between same parallels CE and AYX.

Therefore $ar(ACE) = \frac{1}{2} ar(CYXE)$

$\Rightarrow ar(FCB) = \frac{1}{2} ar(CYXE)$ [Since $\Delta ACE \cong \Delta BCF$, part (iv)]

$\Rightarrow ar(CYXE) = 2ar(FCB)$.

(vi)

Square $ACFG$ and $\Rightarrow BCF$ have the same base CF and are between same parallels CF and BAG.

Therefore $ar(BCF) = \frac{1}{2} ar(ACFG)$

$\Rightarrow \frac{1}{2} ar(CYXE) = \frac{1}{2} ar(ACFG)$ [Using part (v)]

$\Rightarrow ar(CYXE) = ar(ACFG)$

(vii)

From part (iii) and (vi) we have

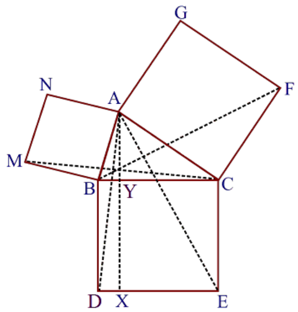
$ar(BYXD) = ar(ABMN)$

and

$ar(CYXE) = ar(ACFG)$

On adding we get

$ar(BYXD) + ar(CYXE) = ar(ABMN) + ar(ACFG) ar(BCED) = ar(ABMN) + ar(ACFG)$



#464041

Topic: Properties of Triangles

A circular park of radius $20m$ is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk to each other. Find the length of the string of each phone.

Solution

Let Ankur be represented as A, Syed as S and David as D.

The boys are sitting at equal distance.

Hence, $\triangle ASD$ is an equilateral triangle.

Let the radius of the circular park be r meters.

$\therefore OS = r = 20$m.

Let the length of each side of $\triangle ASD$ be x meters.

Draw $AB \perp SD$

$\therefore SB = BD = \dfrac{1}{2}SD = \dfrac{x}{2}$ m

In $\triangle ABS$, $\angle B = 90^o$

By Pythagoras theorem,

$AS^2 = AB^2 + BS^2$

$\therefore AB^2 = AS^2 - BS^2$

$= x^2 - \left(\dfrac{x}{2}\right)^2 = \dfrac{3x^2}{4}$

$\therefore AB = \dfrac{\sqrt{3}x}{2}$ m

Now, $AB = AO + OB$

$OB = AB - AO$

$OB = \left(\dfrac{\sqrt{3}x}{2} - 20\right)$ m

In $\triangle OBS$,

$OS^2 = OB^2 + SB^2$

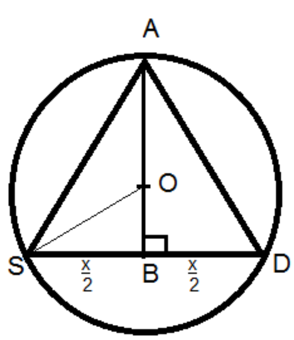
$20^2 = \left(\dfrac{\sqrt{3}x}{2} - 20\right)^2 + \left(\dfrac{x}{2}\right)^2$

$400 = \dfrac{3}{4}x^2 + 400 - 2(20)(\dfrac{\sqrt{3}x}{2}) + \dfrac{x^2}{4}$

$0 = x^2 - 20\sqrt{3}x$

$\therefore x = 20\sqrt{3}$m

Length of string of each phone is $20\sqrt{3}$m.



#464081
Topic: Properties of Triangles

Construct an equilateral triangle, given its side $= 3$ and justify the construction.

Solution

Let equilateral triangle be ABC

Draw $AB = 5CM$

Taking A and B as centers, radius $= AB = 5cm$

draw two areas intersecting each other at A

join AB and AC

for justification just measure the length of each sides