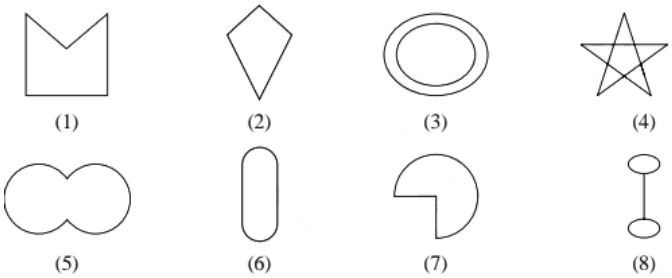
**NCERT Solutions for Class 8 Subject-wise**

- Class 8 Maths
- Class 8 Science – Physics
- Class 8 Science – Biology
- Class 8 Science – Chemistry
- Class 8 Social Science – History
- Class 8 Geography
- Class 8 General Knowledge
- Class 8 Civics

**#463354**

**Topic:** Polygons




Given here are some figures:

Classify each of them on the basis of the following

(a) Simple curve

(b) Simple closed curve

(c) Polygon

(d) Convex polygon

(e) Concave polygon

**Solution**

A curve is said to be simple if it does not cross itself.

(a) Simple curve $\rightarrow$ 1, 2, 5, 6, 7

A simple closed curve is that curve which forms a path which starts and ends at the same point.

(b) Simple closed curve $\rightarrow$ 1, 2, 5, 6, 7

A polygon is a two dimensional closed figure which consists of sides and vertices.

(c) Polygon $\rightarrow$ 1, 2

A polygon in which the measure of each interior angle is less than $180^o$ is called as a convex polygon.

(d) Convex polygon $\rightarrow$ 2

A polygon in which the measure of one interior angle is greater than $180^o$ is called as a concave polygon.

(e) Concave polygon $\rightarrow$ 1

**#463355**

**Topic:** Polygons

How many diagonals does each of the following have?

(a) A convex quadrilateral

(b) A regular hexagon

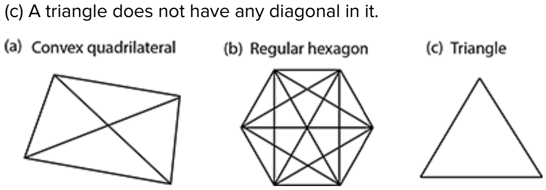
(c) A triangle

**Solution**

(a) There are $2$ diagonals in a convex quadrilateral.

(b) There are $9$ diagonals in a regular hexagon.

(c) A triangle does not have any diagonal in it.



**#463357**

**Topic:** Polygons

| Figure |  |  |  |  |
|---|---|---|---|---|
| Side | 3 | 4 | 5 | 6 |
| Angle sum | 180° | $2\times180°$ $=(4\text{-}2)\times180°$ | $3\times180°$ $=(5\text{-}2)\times180°$ | $4\times180°$ $=(6\text{-}2)\times180°$ |

Examine the table.

(Each figure is divided into triangles and the sum of the angles deduced from that.)

What can you say about the angle sum of a convex polygon with number of sides?

(a) $7$ (b) $8$ (c) $10$ (d) $n$

**Solution**

As per above table, convex polygon with $4$ sides have angle sum $(4-2)\times180^o$

(a)

$\therefore (7-2)\times180^o=900^o$ is the angle sum of convex polygon with $7$ sides

(b)

$(8-2)\times180^o=1080^o$ is the angle sum of convex polygon with $8$ sides

(c)

$(10-2)\times180^o=1440^o$ is the angle sum of convex polygon with $10$ sides

(d) For $n$ sides

$(n-2)\times180^o$ is the angle sum of convex polygon with $n$ sides

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**#463359**

**Topic:** Polygons

What is a regular polygon?

State the name of a regular polygon of

(i) $3$ sides (ii) $4$ sides (iii) $6$ sides

**Solution**

Regular Polygon: A polygon with all sides and angles equal.

(i) $3$ sides: Equilateral triangle

(ii) $2$ sides: Square

(iii) $6$ sides: Regular Hexagon

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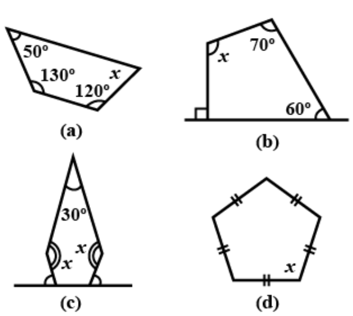
**#463360**

**Topic:** Polygons




Find the angle measure of $x$ in the following figures.

**Solution**

(a)

Sum of all interior angles of quadrilateral $= 360^0$

$\therefore 50 + 130 + 120 + x = 300$

$x = 60^0$

(b)

$90 + a = 180$

$a = 90$

Sum of all angles of quad $= 360$

$x + 70 + 60 + 90 = 360$

$x = 140^0$

(c)

$70 + a = 180$ (Linear pair)

$a = 110$

$60 + b = 180$ (Linear pair)

$b = 120$

Sum of measures of all angles of pentagon $= 540^0$

$\therefore 120 + 110 + x + x + 30 = 540$

$2x = 280$

$x = 140^0$

(d)

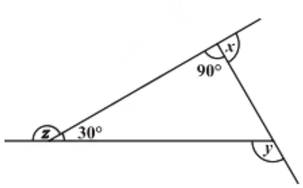
Sum of all angles of pentagon $540$

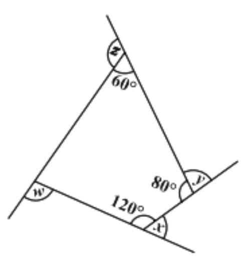
Given all angles are equal

$\therefore 5x = 540$

$x = 108^0$

**#463361**
**Topic:** Polygons




(a) Find $x + y + z$

(b) Find $x + y + z + w$

**Solution**

(a)

$x + 90 = 180$ (Linear pair)

$x = 90$

$z + 30 = 180$, $z = 150$, $y = 90 + 30$ (Exterior angle)

$y = 120$

$x + y + z = 90 + 120 + 150 = 300^0$

(b)

Sum of all measure of all interior angles of quad $= 360$

$a + 60 + 80 + 120 = 360$

$a = 100$

$x + 120 = 180$ (Linear pair)

$x = 60$

$y + 80 = 180$, $y = 100$

$z + 60 = 180$, $z = 120$

$w + 100 = 180$, $w = 80$

Sum of measure of all interior angles

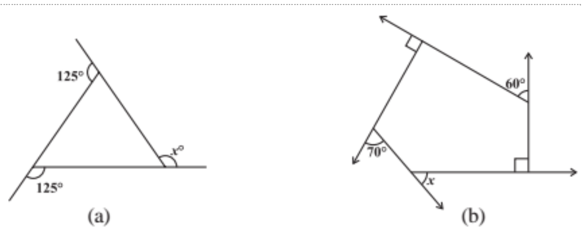
$= x + y + z + w$

$= 60 + 100 + 100 + 80$

$= 360^0$

#463362

**Topic:** Polygons




Find $x$ in the following figures.

**Solution**

Sum of all exterior angles of any polygon $= 360^\circ$

(a)

$125^\circ + 125^\circ + x = 360^\circ$

$x = 360^\circ - 250^\circ$

$x = 110^\circ$

(b)

$60^0 + 90^0 + 70^0 + x + 90^0 = 360^0$

$x = 50^0$

#463363

**Topic:** Polygons

Find the measure of each exterior angle of a regular polygon of

(i) 9 sides (ii) 15 sides

**Solution**

Sum of all exterior angles of given polygon $= 360°$

(i) Measure of each ext angle $= \frac{360}{9} = 40°$

(ii) Measure of each ext angle $= \frac{360}{15} = 24°$

**#463365**

**Topic:** Polygons

How many sides does a regular polygon have if the measure of an exterior angle is $24°$?

**Solution**

Sum of all exterior angles $= 360°$

Measure of each exterior angle $= 24°$

$\therefore$ No of sides will be $\Rightarrow \frac{360}{24} = 15$

**#463366**

**Topic:** Polygons

How many sides does a regular polygon have if each of its interior angles is $165°$?

**Solution**

Measure of each interior angle $= 165°$

Measure of each exterior angle $= 180 - 165 = 15°$

Sum of exterior angles of any polygon $= 360°$

Number of sides $= \frac{360}{15} = 24$

**#463367**

**Topic:** Polygons

(a) Is it possible to have a regular polygon with measure of each exterior angle as $22°$?

(b) Can it be an interior angle of a regular polygon? Why?

**Solution**

(a)

Exterior angle $= 22°$

$360°$ is not perfect multiple of $22°$.

It is not possible to have such polygon.

(b)

Interior angle $= 22°$

Exterior angle $= 180 - 22 = 158°$

$158°$ is not perfect multiple of $360°$.

It is not possible to have such polygon.

**#463368**

**Topic:** Polygons

(a) What is the minimum interior angle possible for a regular polygon? Why?

(b) What is the maximum exterior angle possible for a regular polygon?

**Solution**

For a regular polygon, as the number of sides increase, interior angle also increases and exterior angle decreases.

Therefore, polygon with minimum interior angle and maximum exterior angle is an equilateral triangle, as it has a minimum number of sides possible for a polygon i.e., $3$.

(a) Interior angle of equilateral triangle is $60^0$, which is the minimum interior angle possible for the regular polygon.

(b) Exterior angle of an equilateral triangle is $180^0 - 60^0 = 120^0$, which is the maximum exterior angle possible for the regular polygon.

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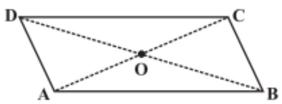
**#463369**

**Topic:** Properties of Quadrilaterals




Given a parallelogram $ABCD$. Complete each statement along with the definition or property used.

(i) $AD =$

(ii) $\angle DCB =$

(iii) $OC =$

(iv) $m\angle DAB + m\angle CDA =$

**Solution**

(i) $AD = BC$

[Opposite sides are equal in parallelogram]

(ii) $\angle DCB = \angle DAB$

[Opposite angles are equal]

(iii) $OC = OA$

[Diagonals bisect each other]

(iv) $m\angle DAB + m\angle CDA = 180°$

[Adjacent angles are supplementary]

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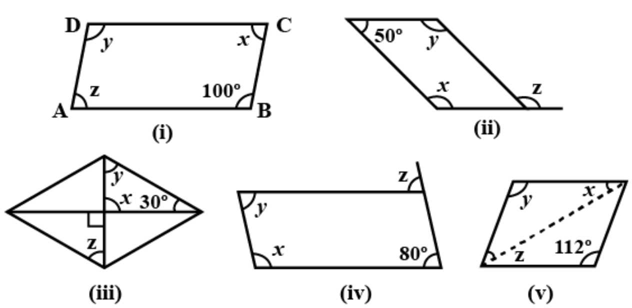
**#463370**

**Topic:** Properties of Quadrilaterals




Consider the following parallelograms. Find the values of the unknowns $x, y, z$

**Solution**

(i) $x + 100 = 180$ [Adjacent angles]

$x = 80$

$z = x = 80^\circ$ [opposite angles are equal]

$y = 100^\circ$

(ii) $50 + y = 180$

$y = 130^\circ$

$x = y = 130^\circ$ and $z = x = 130^\circ$ (corresponding angles)

(iii) $x = 90^\circ$ (vertically opposite angles)

$x + y + 30 = 180^\circ$

$y = 180 - 120 = 60^\circ$

$z = y = 60^\circ$ (Alternate angles)

(iv) $z = 80, y = 80$ (Opposite angles)

$x + y = 180^\circ, x = 180 - 80 \Rightarrow x = 100^\circ$

(v) $y = 112^\circ$ (opposite angles)

$x + y + 40 = 180^\circ$ (Angle sum property)

$x = 180 - 152$

$x = 28^\circ$

$z = x = 28^\circ$ (Alternate angles)

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**#463371**

**Topic:** Properties of Quadrilaterals

Can a quadrilateral $ABCD$ be a parallelogram if

(i) $\angle D + \angle B = 180^\circ$?

(ii) $AB = DC = 8$ cm, $AD = 4$ cm and $BC = 4.4$ cm?

(iii) $\angle A = 70^\circ$ and $\angle C = 65^\circ$?

**Solution**

(i) $\angle D + \angle B = 180^\circ$

It may or may not be possible. the sum of measure of adjacent angles should be $180^\circ$.

(ii) $AB = DC = 8$ cm, $AD = 4$ cm and $BC = 4.4$ cm

No, opposite sides are of different lengths.

(iii) $\angle A = 70^\circ$ and $\angle C = 65^\circ$

No, opposite angles should be equal.

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**#463372**

**Topic:** Properties of Quadrilaterals

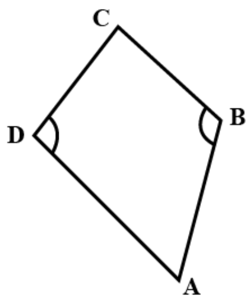
Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure.

**Solution**

The perfect answer would be KITE

$\angle B = \angle D$

$\angle A \neq \angle C$ (Its not a parallelogram)




#463373

**Topic:** Properties of Quadrilaterals

The measures of two adjacent angles of a parallelogram are in the ratio $3:2$. Find the measure of each of the angles of the parallelogram.

**Solution**

Let $\angle A = 3x$ and $\angle B = 2x$

$\angle A + \angle B = 180$

(Adjacent angle : Supplementary)

$3y + 2x \Rightarrow 5x = 180 \Rightarrow x = \frac{180}{5}$

$x = 36°$

$\angle A = \angle C = 3x = 3 \times 36$

$= 108°$

$\angle B = \angle D = 2x = 2 \times 36 = 72°$

#463374

**Topic:** Properties of Quadrilaterals

Two adjacent angles of a parallelogram have equal measure. Find the measure of each of the angles of the parallelogram.

**Solution**

Let the parallelogram be $\square ABCD$.

Sum of adjacent angles $= 180°$

$\angle A + \angle B = 180°$

$2\angle A = 180°$ (Given $\angle A = \angle B$)

$\angle A = 90°$

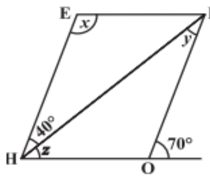
$\angle A = \angle B = 90°$

$\angle B = \angle D = 90°$

All angles are equal to $90°$

#463375

**Topic:** Properties of Quadrilaterals




The above figure is a parallelogram. Find the angle measures $x$, $y$ and $z$. State the properties you use to find them.

**Solution**

$y = 40°$ (Alternate angles)

$70 = z + 40$ (Corresponding angles)

$z = 30°$

$x + (z + 40) = 180°$ (Adjacent angles)

$x + 70 = 180$

$x = 110°$

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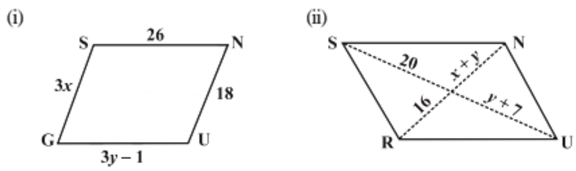
**#463376**

**Topic:** Properties of Quadrilaterals




The following figures GUNS and RUNS are parallelograms. Find $x$ and $y$.

(Lengths are in cm)

**Solution**

(i) Length of opposite sides are equal

$GU = SN$

$3y - 1 = 26$

$y = \frac{27}{3} = 9$

$GS = NU \Rightarrow 3x = 18$

$x = 6$

$y = 9$

(ii) Diagonals of parallelogram bisect each other

$y + 7 = 20$, $y = 13$

$x + y = 16$, $x + 13 = 16$

$x = 3$

$y = 13$

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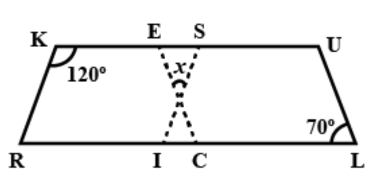
**#463379**

**Topic:** Properties of Quadrilaterals




In the above figure both RISK and CLUE are parallelograms. Find the value of $x$.

**Solution**

In parallelogram RISK,

$\angle RKS + \angle ISK = 180^\circ$

$120^\circ + \angle ISK = 180^\circ$

$\angle ISK = 60^\circ$

Adjacent angles are supplementary

Also, opposite angles are equal

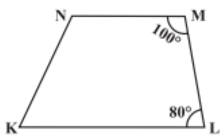
In parallelogram CLUE, $\angle ULC = \angle CEU = 70^\circ$

$x + 60^\circ + 70^\circ = 180^\circ$

$x = 50^\circ$

**#463380**

**Topic:** Properties of Quadrilaterals




Explain how this figure is a trapezium. Which of its two sides area parallel?

**Solution**

Given two line will parallel to each other

$\angle NML + \angle MLK = 180^\circ$

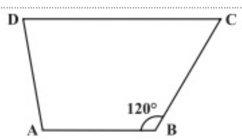
Hence $NM \parallel LK$

As quadrilateral KLMN a pair of parallel lines

$\therefore$ It is a trapezium.

**#463383**

**Topic:** Properties of Quadrilaterals




Find $m\angle C$ in fig if $\bar{AB} \parallel \bar{DC}$

**Solution**

$AB \parallel DC$

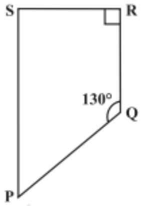
$\angle B + \angle C = 180^\circ$ (Angles on same side of transversal are supplementry)

$\angle C + 120 = 180^\circ$

$\angle C = 60^\circ$

**#463384**

**Topic:** Polygons




Find the measure of $\angle P$ and $\angle S$ if $\bar{SP} \parallel \bar{RQ}$ & in Fig.

(If you find $m\angle R$, is there more than one method to find $m\angle P$?)

**Solution**

$\angle P = 180 - (\angle Q)\angle P = 180 - (130)\angle P = 180 - (130) = 50^0$

Given, $PS$ is parallel to $RQ$.

So, $\angle P \& \angle Q$ form co-interior angles, which are also supplementary.

Now, using the sum of all angles of a quadrilateral $= 360$ degrees, we can find $\angle S$.

$\angle P + \angle R + \angle S + \angle Q = 36050 + 90 + \angle S + 130 = 360\angle S + 270 = 360\angle S = 90^0$

$SP \parallel RQ$ ...given

$\therefore \angle P + \angle Q = 180^o$ ....interior angles

$\therefore \angle P + 130^o = 180^o$

$\therefore \angle P = 50^o$

m$\angle R = 90^o$ ...given

In $\square PQRS$,

$\angle P + \angle Q + \angle R + \angle S = 360^o$ ...Angle sum property of quadrilateral

$\therefore 50^o + 130^o + 90^o + \angle S = 360^o$

$\therefore \angle S = 90^o$

**#463385**
**Topic:** Polygons

State whether True or False.

(a) All rectangles are squares.

(b) All rhombuses are parallelograms.

(c) All squares are rhombuses and also rectangles.

(d) All squares are not parallelograms.

(e) All kites are rhombuses.

(f) All rhombuses are kites.

(g) All parallelograms are trapeziums.

(h) All squares are trapeziums.

**Solution**

(a) False

All squares are rectangles.

(All sides of rectangle are not equal).

(b) True

Opposite sides are equal and parallel.

(c) True

All sides of the rhombus are equal

$\therefore$ All squares are rhombuses.

All squares are also rectangles as each interval angle measure $90^\circ$

(d) False

All squares are parallelogram as opposite sides are equal and parallel.

(e) False

Kite has different lengths.

(f) True

Rhombus also has two distinct consecutive pairs of sides of equal length.

(g) True

Parallelogram pair of parallel sides.

(h) True

All squares have a pair of parallel sides.

**#463386**
**Topic:** Polygons

Identify all the quadrilaterals that have

(a) four sides of equal length

(b) four right angles

**Solution**

(a) Quadrilaterals that have four sides of equal length are Rhombus and squares.

(b) Quadrilaterals that have four right angles are Rectangle and squares.

**#463387**
**Topic:** Polygons

Explain how a square is:

(i) a quadrilateral (ii) a parallelogram (iii) a rhombus (iv) a rectangle

**Solution**

(a) A quadrilateral has four sides

(b) A parallelogram's opposite sides are parallel.

(c) A rhombus's four sides are of same length

(d) A rectangle's interior angles are equal to $90^\circ$.

All of these properties are followed by Square.

#463388

**Topic:** Polygons

Name the quadrilaterals whose diagonals.

(i) bisect each other

(ii) are perpendicular bisectors of each other

(iii) are equal

**Solution**

(i) Bisects each other: Diagonals of a parallelogram, rhombus, square and rectangle.

(ii) Are perpendicular bisectors of each other: Diagonals of rhombus and square $\rightarrow$ perpendicular bisectors.

(iii) Are equal: Diagonals of rectangle and square are equal.

#463389

**Topic:** Polygons

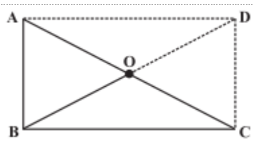
Explain why a rectangle is a convex quadrilateral.

**Solution**

The rectangle is a convex quadrilateral because of two diagonals which lie in the interior of the rectangle.

#463390

**Topic:** Properties of Quadrilaterals




ABC is a right-angled triangle and O is the mid point of the side opposite to the right angle. Explain why O is equidistant from A, B and C.

(The dotted lines are drawn additionally to help you)

**Solution**

$AD = BC$ and $AB = DC$

$ABCD$ is a rectangle as opposite sides are equal and parallel to each other and interior angles are of $90°$.

In rectangle, diagonals are of equal length and also these bisect each other.

$AO = OC = BO = OD$

O is equidistant from $A$, $B$ and $C$.

#463875

**Topic:** Properties of Quadrilaterals

If the diagonals of a parallelogram are equal, then show that it is a rectangle.

**Solution**

Given: In parallelogram $ABCD$, $AC = BD$

To prove : Parallelogram $ABCD$ is rectangle.

Proof : in $\triangle ACB$ and $\triangle BDA$

$AC = BD$ | Given

$AB = BA$ | Common

$BC = AD$ | Opposite sides of the parallelogram ABCD

$\triangle ACB \cong \triangle BDA$ | $SSS$ Rule

$\therefore \angle ABC = \angle BAD...(1)$ CPCT

Again $AD \parallel$ | Opposite sides of parallelogram $ABCD$

$AD \parallel BC$ and the traversal $AB$ intersects them.

$\therefore \angle BAD + \angle ABC = 180°$ ...(2) _ Sum of consecutive interior angles on the same side of the transversal is $180°$

From (1) and (2) ,

$\angle BAD = \angle ABC = 90°$

$\therefore \angle A = 90°$ and $\angle C = 90°$

Parallelogram $ABCD$ is a rectangle.

In $\triangle ACB$ and $\triangle BDA$

$AC = BD$ | (Provided in the question)

$AB = BA$ |

$BC = AD$ | Opposite sides of the parallelogram ABCD

$\triangle ACB \cong \triangle BDA$\$

$\therefore \angle ABC = \angle BAD...(1)$ CPCT

$AD \parallel$ | Opposite sides of parallelogram $ABCD$

$AD \parallel BC$ and the traversal $AB$ intersects them.

$\therefore \angle BAD + \angle ABC = 180°$ ...(2) _ Sum of consecutive interior angles on the same side of the transversal is $180°$

From (1) and (2) ,

$\angle BAD = \angle ABC = 90°$

$\therefore \angle A = 90°$ and $\angle C = 90°$

Parallelogram $ABCD$ is a rectangle.

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**#463878**

**Topic:** Properties of Quadrilaterals

Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

**Solution**

Given:

$\square ABCD$ is a quadrilateral.

diag $AC =$ diag $BD$, intersecting at $E$.

$AC$ and $BD$ are perpendicular bisectors of each other

$\therefore \angle E = 90^o$

To prove:

$\square ABCD$ is a square.

Solution:

In $\triangle ABE$ and $\triangle ADE$.

$BE = DE$ ....given

$AE \cong AE$ ...common side

$\angle AEB \cong \angle AED$ ....each $90^o$

$\therefore \triangle ABE \cong \triangle ADE$ ...SAS test of congruence

$\therefore AB = AD$ ...c.s.c.t. ....(1)

Similarly, we can prove $\triangle ABE \cong \triangle CBE$

$\therefore AB = CB$ ....c.s.c.t. ....(2)

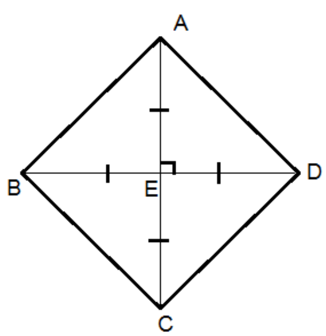
And, $\triangle ADE \cong \triangle CDE$

$\therefore AD = CD$ ....c.s.c.t. ....(3)

$\therefore$ From (1), (2) and (3),

$AB = CB = CD = AD$

$\therefore \square ABCD$ is a square ....By definition



Consider a quadrilateral $ABCD$

with diagonals $AC$ and $BD$ which intersect at $O$.

In $\triangle AOB$ and $\triangle BOC$,

$\angle AOB = \angle BOC$ (Both are $90^o$)

$OA = OC$ (Diagonal $AC$ is bisected at $O$)

$OB = OB$ (common side)

By SAS congruence,

$\triangle AOB \cong \triangle BOC$

So, $AB = CB$ (By CPCT) ..... (1)

In $\triangle AOB$ and $\triangle COD$

$\angle AOB = \angle COD$ (Both are $90^o$)

$OA = OC$ (Diagonal $AC$ is bisected at $O$)

$OB = OD$ (Diagonal $BD$ is bisected at $O$)

By SAS congruence,

$\triangle AOB \cong \triangle COD$

$AB = CD$ (By CPCT) ....(2)

Similarly, we can prove $\triangle AOB \cong \triangle AOD$

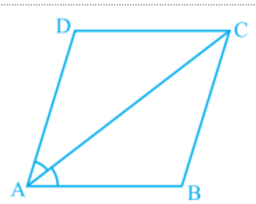
$AB = AD$ (By CPCT) .... (3)

$AB = BC = CD = AD$

So, ABCD is a is a square.

**#463879**

**Topic:** Properties of Quadrilaterals




Diagonal $AC$ of a parallelogram $ABCD$ bisects $\angle A$. Show that

(i) It bisects $\angle C$ also,

(ii) $ABCD$ is a rhombus

**Solution**

$\square ABCD$ is a parallelogram ....given

side $AD \cong$ side $BC$ ...opposite sides of a parallelogram are congruent ....(1)

In $\triangle ADC$ and $\triangle ABC$,

side $AD \cong$ side $BC$ ....from (1)

$\angle DAC = \angle BAC$ .....given

side $AC \cong$ side $AC$ ....common side

$\triangle ADC \cong \triangle ABC$ .....SAS test of congruence

$\therefore \angle DCA = \angle BCA$ ....c.a.c.t. ....(2)

$\therefore$ Diag $AC$ is bisector of $\angle C$.

We know, opposite angles of a parallelogram are congruent.

$\angle DAB = \angle BCD$

$\therefore \frac{1}{2}\angle DAB = \frac{1}{2}\angle BCD$

$\therefore \angle BAC = \angle BCA$ ....(3)

In $\triangle ABC$,

$\angle BAC = \angle BCA$ ....from (3)

$\therefore BC = AB$ ....Converse of isosceles triangle theorem

Similarly, we can prove $AD = DC$.

Since, the adjacent sides of a parallelogram are equal,

$\square ABCD$ is a rhombus.

**#463880**

**Topic:** Properties of Quadrilaterals

ABCD is a rhombus. Show that diagonal $AC$ bisects $\angle A$ as well as $\angle C$ and diagonal $BD$ bisects $\angle B$ as well as $\angle D$

**Solution**

In $\triangle ABD$ and $\triangle BCD$,

$AB = BC$ ....Sides of a rhombus

$AD = CD$ ....Sides of a rhombus

$AC = AC$ ....Common side

$\triangle ABD \cong \triangle BCD$ ...SSS test of congruence

$\therefore \angle ABD = \angle CBD$ ....C.P.C.T.

$\therefore \angle ADB = \angle CDB$ ....C.P.C.T.

So, Diagonal $BD$ bisects $\angle B$ and $\angle D$ of the rhombus $ABCD$.

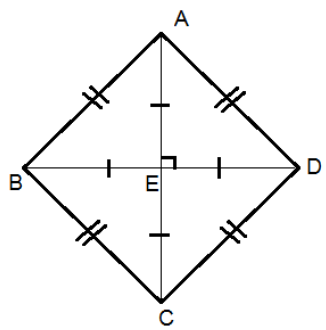
Similarly we can prove,

$\triangle ABC \cong \triangle ADC$

$\therefore \angle BAC = \angle DAC$ ....C.P.C.T.

$\therefore \angle BCA = \angle DCA$ ....C.P.C.T

So, Diagonal $AC$ bisects $\angle A$ and $\angle C$ of the rhombus $ABCD$.




**#463882**

**Topic:** Properties of Quadrilaterals

$ABCD$ is a rectangle in which diagonal $AC$ bisects $\angle A$ as well as $\angle C$. Show that:

(i) $ABCD$ is a square (ii) diagonal $BD$ bisects $\angle B$ as well as $\angle D$

**Solution**

$\square ABCD$ is a rectangle ...given

$\therefore$ All the angles are equal to $90^\circ$ ...by definition

$\therefore \angle A = \angle C$ ...($AC$ bisects $\angle A$ as well as $\angle C$)

So, $\dfrac{\angle A}{2} = \dfrac{\angle C}{2}$

So, $\angle x = \angle y = \angle z = \angle w$ ....(i)

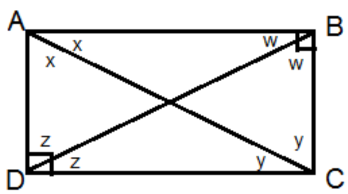
In $\triangle ADC$,

$AD = CD$ ....from (i) and converse of isosceles triangle theorem

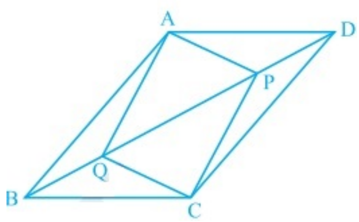
So, $\square ABCD$ is a square.

(ii) Diagonals bisect the angles in a square.

$\Rightarrow BD$ bisects $\angle B$ as well as $\angle D$.




**#463883**
**Topic:** Properties of Quadrilaterals




In parallelogram $ABCD$, two point $P$ and $Q$ are taken on diagonal $BD$ such that $DP = BQ$. Show that

(i) $\triangle APD \cong \triangle CQB$

(ii) $AP = CQ$

(iii) $\triangle AQB \cong \triangle CPD$

(iv) $AQ = CP$

(v) $APCQ$ is a parallelogram

**Solution**

Construction: Join $AC$ to meet $BD$ in $O$.

Therefore, $OB = OD$ and $OA = OC$ ...(1)

(Diagonals of a parallelogram bisect each other)

But $BQ = DP$ ...given

$\therefore OB - BQ = OD - DP$

$\therefore OQ = OP$ ....(2)

Now, in $\square APCQ$,

$OA = OC$ ....from (1)

$OQ = OP$ ....from (2)

$\therefore \square APCQ$ is a parallelogram.

(i) In $\triangle APD$ and $\triangle CQB$,

$AD = CB$ ....opposite sides of a parallelogram

$AP = CQ$ ....opposite sides of a parallelogram

$DP = BQ$ ...given

$\triangle APD \cong \triangle CQB$ ...By SSS test of congruence

(ii) $\therefore AP = CQ$ ...c.s.c.t.

(iv) and $AQ = CP$ ...c.s.c.t. ...(3)

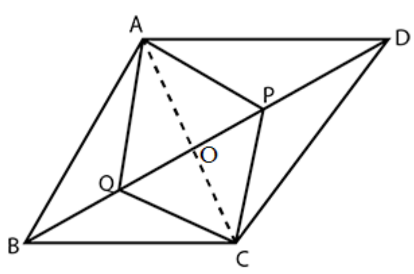
(iii) In $\triangle AQB$ and $\triangle CPD$,

$AB = CD$ ....opposite sides of a parallelogram

$AQ = CP$ ...from (3)

$BQ = DP$ ...given

$\therefore \triangle AQB \cong \triangle CPD$ ....By SSS test of congruence




**#463890**
**Topic:** Properties of Quadrilaterals

$ABCD$ is a rhombus and $P$, $Q$, $R$ and $S$ are the mid-points of the sides $AB$, $BC$, $CD$ and $DA$ respectively. Show that the quadrilateral $PQRS$ is a rectangle

**Solution**

In $\triangle ABC$, $P$ and $Q$ are the mid- points of $AB$ and $BC$.

$PQ \parallel AC$ and by using mid - point theorem

$PQ = \frac{1}{2}AC$.

Similarly, in $\triangle ADC$, $R$ and $S$ are the mid- points of $CD$ and $AD$.

$SR \parallel AC$ and by using mid point theorem

$SR = \frac{1}{2}AC$

From (1) and (2), we get

$PQ \parallel RS$ and $PQ = SR$

Now, in quadrilateral $PQRS$ its one pair of opposite sides $PQ$ and $SR$ is equal and parallel.

Therefore,

$PQRS$ is a parallelogram

$AB = BC$ (Sides of a rhombus)

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}BC$

$PB = BQ$

$\angle 3 = \angle 4$

Now, in $\triangle APS$ and $\triangle CQR$, we have

$AP = CQ$ (Halves of equal sides $AB$, $BC$)

$AS = CR$ (Halves of equal sides $AD$, $CD$)

$PS = QR$ (Opp. sides of parallelogram $PQRS$)

Therefore, $\triangle APS \cong \triangle CQR$ | using SSS Congruency Theorem

$\angle 1 = \angle 2$ (Corresponding parts of congruent triangles are equal)

Now , $\angle 1 + \angle SPQ + \angle 3 = 180^o$ (Linear pair axiom)

Therefore, $\angle 1 + \angle SPQ + \angle 3 = \angle 2 + \angle PQR + \angle 4$

But, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

Therefore,

$\angle SPQ = \angle PQR$ .....(3)

Since, $SP \parallel RQ$ and $PQ$ intersects them

Therefore, $\angle SPQ + \angle PQR = 180^o$....(4) (Since consecutive interior angles are supplementary)

From (3) and (4), we get

$\angle PQR + \angle PQR = 180^o$

$2\angle PQR = 180^o$

$\angle PQR = 90^o$

$\angle SPQ = \angle PQR = 90^o$

Thus, $PQRS$ is a parallelogram whose one angle $\angle SPQ = 90^o$.

Hence $PQRS$ is a rectangle.

**#463891**

**Topic:** Properties of Quadrilaterals

$ABCD$ is a rectangle and $P$, $Q$, $R$ and $S$ are mid-points of the sides $AB$, $BC$, $CD$ and $DA$ respectively. Show that the quadrilateral $PQRS$ is a rhombus

**Solution**

In $\triangle ABC$, $P$ and $Q$ are the mid-points of sides $AB$ and $BC$.

Using Mid-point Theorem,

$PQ \parallel AC$ and $PQ = \dfrac{1}{2}AC$ ...(1)

Similarly,

In $\triangle ADC$, $R$ and $S$ are the mid-points of sides $CD$ and $AD$.

Using Mid-point Theorem

$SR \parallel AC$ and $SR = 12AC$ ...(2)

From (1) and (2), we get

$PQ \parallel SR$ and $PQ = SR$ ......(3)

Now,

In quad. $PQRS$, its one pair of opposite side $PQ$ and $SR$ is parallel and equal.

Therefore,

$PQRS$ is a parallelogram ... (4)

Now,

$AD = BC$ (Opp. sides of rect. $ABCD$)

$\Rightarrow \dfrac{1}{2}AD = \dfrac{1}{2}BC$

$\Rightarrow AS = BQ$

In $\triangle APS$ and $\triangle BPQ$ , we have

$AP = BP$ (Since $P$ is the mid- point of $AB$)

$\angle PAS = \angle PBQ$ (Each $= 90^o$)

$AS = BQ$

Therefore,

$\triangle APS \cong \triangle BPQ$ (SAS congruence axiom)

$\Rightarrow PS = PQ$ (Corresponding parts of congruent triangles are equal) ... (5)

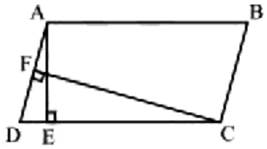
From (4) and (5), we get,

$PQ = QR = RS = PS$

$PQRS$ is a rhombus.

**#463915**

**Topic:** Properties of Quadrilaterals




In fig, $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$. If $AB = 16\ cm$, $AE = 8\ cm$ and $CF = 10\ cm$, find $AD$

**Solution**

Given,

$AB = CD = 16$ cm ....Opposite sides of a parallelogram

$CF = 10$ cm and $AE = 8$ cm

Now,

Area of parallelogram $=$ Base $\times$ Altitude

$= CD \times AE = AD \times CF$

$16 \times 8 = AD \times 10$

$AD = \dfrac{128}{10}$ cm

$AD = 12.8$ cm

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**#463916**

**Topic:** Properties of Quadrilaterals

If $E$, $F$, $G$ and $H$ are respectively the mid-points of the sides of a parallelogram $ABCD$, show that $ar(EFGH) = \dfrac{1}{2} ar(ABCD)$

**Solution**

Given:

$E$, $F$, $G$ and $H$ are respectively the mid-points of the sides of a parallelogram $ABCD$.

To Prove:

$ar(EFGH) = \frac{1}{2} ar(ABCD)$

Construction:

$H$ and $F$ are joined.

Proof:

$AD \parallel BC$ and $AD = BC$ (Opposite sides of a parallelogram)

$\Rightarrow \frac{1}{2} AD = \frac{1}{2} BC$

Also,

$AH \parallel BF$ and and $DH \parallel CF$

$\Rightarrow AH = BF$ and $DH = CF$ | $H$ and $F$ are mid points

Thus,

$ABFH$ and $HFCD$ are parallelograms.

Now,

$\triangle EFH$ and llgm $ABFH$ lie on the same base $FH$ and between the same parallel lines $AB$ and $HF$.

$\therefore$ Area of $EFH = \frac{1}{2} ar(ABFH)$ --- (i)

Also,

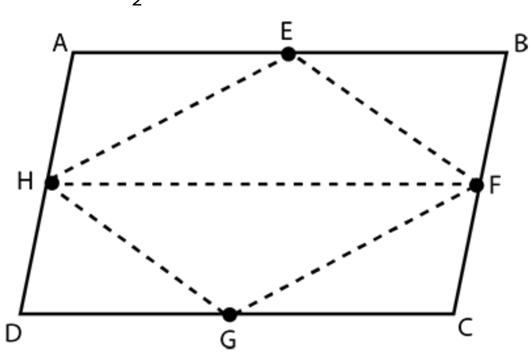
Area of $GHF = \frac{1}{2} ar(HFCD)$ --- (ii)

Adding (i) and (ii),

Area of $\triangle EFH +$ area of $\triangle GHF = \frac{1}{2} ar(ABFH) + \frac{1}{2} ar(HFCD)$

$\Rightarrow$ Area of $EFGH =$ Area of $ABFH$

$\Rightarrow ar(EFGH) = \frac{1}{2} ar(ABCD)$




**#463925**

**Topic:** Properties of Quadrilaterals

Show that the diagonals of a parallelogram divide it into four triangles of equal area.

**Solution**

$O$ is the mid point of $AC$ and $BD$. (diagonals of bisect each other)

In $\triangle ABC$, $BO$ is the median.

$\therefore ar(AOB) = ar(BOC)$--- (i)

Also,

In $\triangle BCD$, $CO$ is the median.

$\therefore ar(BOC) = ar(COD)$ --- (ii)

In $\triangle ACD$, $OD$ is the median.

$\therefore ar(AOD) = ar(COD)$--- (iii)

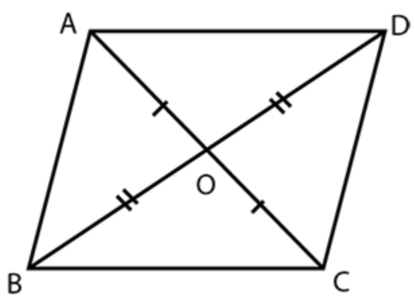
In $\triangle ABD$, $AO$ is the median.

$\therefore ar(AOD) = ar(AOB)$--- (iv)

From equations (i), (ii), (iii) and (iv),

$ar(BOC) = ar(COD) = ar(AOD) = ar(AOB)$

So, the diagonals of a parallelogram divide it into four triangles of equal area.

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**#463934**

**Topic:** Properties of Quadrilaterals

Diagonals $AC$ and $BD$ of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at $O$. Prove that $ar(AOD) = ar(BOC)$

**Solution**

Triangles $ABC$ and $ABD$ are on the same base $AB$ and between the same parallels $AB$ and $DC$.

Therefore,

$ar(ABD) = ar(ABC)$

Subtract area of $\triangle AOB$ from both the sides.

$ar(ABD) - ar(AOB) = ar(ABC) - ar(AOB)$

$\Rightarrow ar(AOD) = ar(BOC)$

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**#463940**

**Topic:** Properties of Quadrilaterals

$ABCD$ is a trapezium with $AB \parallel DC$. A line parallel to $AC$ intersects $AB$ at $X$ and $BC$ at $Y$. Prove that $ar(\triangle ADX) = ar(\triangle ACY)$

**Solution**

Given : $ABCD$ is a trapezium with $AB \parallel DC$ and $XY \parallel AC$.

Join $YA$, $XC$ and $XD$.

Triangles $ACX$ and $ACY$ have same base $AC$ and are between same parallels $AC$ and $XY$

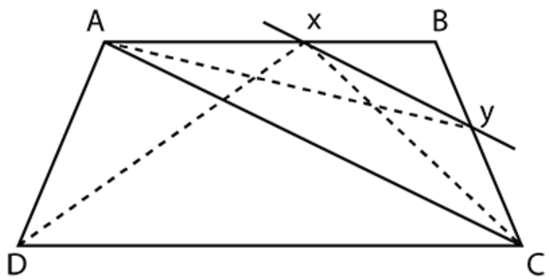
So, $ar(\triangle ACX) = ar(\triangle ACY)$ ......(i)

Triangles $ACX$ and $ADX$ have same base $AX$ and are between same parallels $AB$ and $DC$.

So, $ar(\triangle ACX) = ar(\triangle ADX)$ ......(ii)

From (i) and (ii),

$ar(\triangle ADX) = ar(\triangle ACY)$




#463942

**Topic:** Properties of Quadrilaterals

Diagonals $AC$ and $BD$ of a quadrilateral $ABCD$ intersect at $O$ in such a way that $ar(\triangle AOD) = ar(\triangle BOC)$. Prove that $ABCD$ is a trapezium

**Solution**

Given :

For a quadrilateral $ABCD$, diagonals $AC$ and $BD$ intersect at $O$

And $ar(\triangle AOD) = ar(\triangle BOC)$

To prove:

$ABCD$ is a trapezium.

Adding $ar(ODC)$ on both sides ,

$ar(\triangle AOD) + ar(\triangle ODC) = ar(\triangle BOC) + ar(\triangle ODC)$

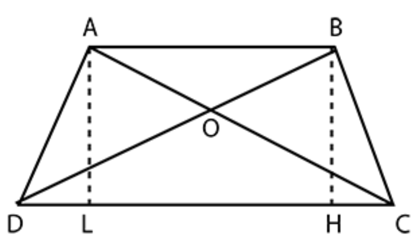
$\Rightarrow ar(\triangle ADC) = ar(\triangle BDC)$

$\Rightarrow \frac{1}{2} \times DC \times AL = \frac{1}{2} \times DC \times BH$

$\Rightarrow AL = BH$

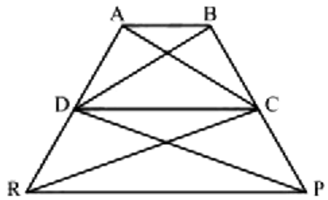
$\Rightarrow AB \parallel DC$

So, $ABCD$ is a trapezium.




#463943

**Topic:** Properties of Quadrilaterals




In fig, $ar(DRC) = ar(DPC)$ and $ar(BDP) = ar(ARC)$. Show that both the quadrilaterals $ABCD$ and $DCPR$ are trapeziums

**Solution**

Since, $ar(DRC) = ar(DPC)$, we get

$\frac{1}{2} \times DC \times d_1 = \frac{1}{2} \times DC \times d_2$

(where $d_1$ and $d_2$ are perpendicular distance from $R$ to $DC$ and from $P$ to $DC$ respectively.

$\therefore \ d_1 = d_2$

$\Rightarrow$ $DC$ is parallel to $RP$.

So, $DCPR$ is a trapezium.

Given $ar(BDP) = ar(ACR)$

Since, we know that $ar(DRC) = ar(DPC)$, we get

$ar(ACD) = ar(DBC)$.

$\therefore \ \frac{1}{2} \times CD \times d_3 = \frac{1}{2} \times CD \times d_4$

(where $d_3$ and $d_4$ are perpendicular distances from $A$ to $DC$ and from $B$ to $CD$ respectively..

$\therefore \ d_3 = d_4$

$\Rightarrow$ $AB$ is parallel to $DC$.

So, $DCBA$ is trapezium.