#417855
Topic: Cartesian Product

If \( \left( \frac{x}{3} + 1, \frac{y}{3} - \frac{2}{3} \right) = \left( \frac{5}{3}, \frac{1}{3} \right) \) find the values of \( x \) and \( y \)

Solution

It is given that \( \left( \frac{x}{3} + 1, \frac{y}{3} - \frac{2}{3} \right) = \left( \frac{5}{3}, \frac{1}{3} \right) \)

Since the ordered pairs are equal the corresponding element will also be equal

Therefore \( \frac{x}{3} + 1 = \frac{5}{3} \) and \( \frac{y}{3} - \frac{2}{3} = \frac{1}{3} \)

\[ \Rightarrow y = 1 \]
\[ \Rightarrow \frac{x}{3} + 1 = \frac{5}{3} \]
\[ \Rightarrow x = 2 \]
\[ \therefore x = 2 \text{ and } y = 1 \]

#417856
Topic: Cartesian Product

If the set \( A \) has 3 elements and the set \( B = \{3, 4, 5\} \) then find the number of elements in \( A \times B \)?

Solution

It is given that set \( A \) has 3 elements and the elements of set \( B \) are 3, 4, and 5

\[ \Rightarrow \text{Number of elements in set } B = 3 \]

Number of elements in \( A \times B \)

\[ = \text{Number of elements in } A \times \text{Number of elements in } B \]
\[ = 3 \times 3 = 9 \]

Thus the number of elements in \( A \times B \) is 9

#417864
Topic: Cartesian Product

If \( G = \{7, 8\} \) and \( H = \{5, 4, 2\} \), find \( G \times H \) and \( H \times G \).

Solution

\[ G = \{7, 8\} \text{ and } H = \{5, 4, 2\} \]

We know that the Cartesian product of \( P \times Q \) of two non-empty sets \( P \) and \( Q \) is defined as

\[ P \times Q = \{ (p, q) \mid p \in P, q \in Q \} \]

\[ \therefore G \times H = \{ (7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2) \} \]

and \( H \times G = \{ (5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8) \} \)

#417877
Topic: Cartesian Product

State whether each of the following statements are true or false. If the statement is false rewrite the given statement correctly.

(i) \( \text{If } P = \{m, n\} \text{ and } Q = \{n, m\} \text{ then } P \times Q = \{\{m, n\}, \{n, m\}\} \)

(ii) \( \text{If } A \text{ and } B \text{ are non-empty sets then } A \times B \text{ is a non-empty set of ordered pairs } \{x, y\} \text{ such that } x \in A \text{ and } y \in B \)

(iii) \( \text{If } A = \{1, 2\} \text{ and } B = \{3, 4\} \text{ then } A \times (B \cap \emptyset) = \emptyset \)

Solution
(i) Given \( P = \{m, n\} \) and \( Q = \{n, m\} \) then
\[
P \times Q = [(m, n), (m, m), (n, n), (n, m)]
\]
So, the given value of \( P \times Q \) is incorrect.
Hence, the given statement (i) is false.

(ii) If \( A \) and \( B \) are non-empty sets, then \( A \times B \) is a non-empty set of ordered pairs \((x, y)\) such that \( x \in A \) and \( y \in B \)
Hence, the given statement (ii) is true.

(iii) \( A = \{1, 2\} \) and \( B = \{3, 4\} \)
\[
A \times (B \cap \phi) = A \times \phi = \phi
\]
So, (iii) is true.

### 417884
**Topic:** Cartesian Product

If \( A = \{1, 1\} \) then find \( A \times A \).

**Solution**

It is known that for any non-empty set \( A \), \( A \times A \) is defined as
\[
A \times A = \{(a, b) : a, b \in A\}
\]
It is given that \( A = \{1, 1\} \)
\[
A \times A = \{(1, 1), (1, 1), (1, 1), (1, 1), (1, 1), (1, 1), (1, 1), (1, 1), (1, 1), (1, 1), (1, 1), (1, 1), (1, 1)
\]
\[
\}

### 417885
**Topic:** Cartesian Product

If \( A \times B = \{(a, x), (a, y), (b, x), (b, y)\} \) find \( A \) and \( B \).

**Solution**

It is given that \( A \times B = \{(a, x), (a, y), (b, x), (b, y)\} \)

We know that the Cartesian product of two non-empty sets \( P \) and \( Q \) is defined as
\[
P \times Q = \{(p, q) : p \in P, q \in Q\}
\]
\[
A \times B = \{(a, x), (a, y), (b, x), (b, y)\}
\]
\[
\text{Thus} \ A = \{a, b\} \text{ and } B = \{x, y\}
\]

### 417886
**Topic:** Cartesian Product

Let \( A = \{1, 2\} \), \( B = \{1, 2, 3, 4\} \), \( C = \{5, 6\} \) and \( D = \{5, 6, 7, 8\} \) verify that

(i) \( A \times (B \cap C) = (A \times B) \cap (A \times C) \)

(ii) \( A \times C \) is a subset of \( B \times D \)

**Solution**
I) To verify: \( A \times (B \cap C) = (A \times B) \cap (A \times C) \)

We have \( B \cap C = \{1, 2, 3, 4\} \cap \{5, 6\} = \emptyset \)

\[ L.H.S = A \times (B \cap C) = A \times \emptyset = \emptyset \]

\[ A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\} \]

\[ A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\} \]

\[ R.H.S = (A \times B) \cap (A \times C) = \emptyset \]

\[ L.H.S = R.H.S \]

Hence \( A \times (B \cap C) = (A \times B) \cap (A \times C) \)

II) To verify: \( A \times C \) is a subset of \( B \times D \)

\[ A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\} \]

\[ B \times D = \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\} \]

We can observe that all the elements of set \( A \times C \) are the elements of set \( B \times D \)

Therefore \( A \times C \) is a subset of \( B \times D \)

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#417887

**Topic:** Cartesian Product

Let \( A = \{1, 2\} \) and \( B = \{3, 4\} \). Write \( A \times B \) and find how many subsets will \( A \times B \) have? List them.

**Solution**

\[ A = \{1, 2\} \) and \( B = \{3, 4\} \]

\[ A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\} \]

\[ n(A \times B) = 4 \]

We know that if \( n(A) = m \) and \( n(B) = n \) then

\[ \text{number of subsets of } A \times B = 2^{m \times n} \]

Therefore the set \( A \times B \) has \( 2^4 = 16 \) subsets.

These are:

\[ \emptyset, (1, 3), (1, 4), (2, 3), (2, 4), \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{1, 3, 2, 4\}, \{1, 3, 2, 4\}, \{1, 3\}, \{2, 3\}, \{1, 4\}, \{2, 4\}, \{1, 3, 2, 4\}, \{1, 3, 2, 4\}, \{1, 3\}, \{2, 3\}, \{1, 4\}, \{2, 4\}, \{1, 3, 2, 4\} \]

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#417889

**Topic:** Cartesian Product

Let \( A \) and \( B \) be two sets such that \( n(A) = 3 \) and \( n(B) = 2 \). If \( (x, y), (y, z) \) are in \( A \times B \) find \( A \times B \) where \( x, y, \) and \( z \) are distinct elements.

**Solution**

It is given that \( n(A) = 3 \) and \( n(B) = 2 \) and \( (x, y), (y, z) \) are in \( A \times B \)

We know that \( A = \) set of first elements of the ordered pair elements of \( A \times B \)

\[ B = \) set of second elements of ordered pair elements of \( A \times B \]

\[ x, y \) and \( z \) are the elements of \( A \) and \( 1 \) and \( 2 \) are the elements of \( B \)

Since \( n(A) = 3 \) and \( n(B) = 2 \) it is clear that \( A = \{x, y, z\} \) and \( B = \{1, 2\} \)

---

#417891

**Topic:** Cartesian Product

The cartesian product \( A \times A \) has 9 elements among which are found \( \{1, 0\} \) and \( \{0, 1\} \). Find the set \( A \) and the remaining elements of \( A \times A \)

**Solution**

The cartesian product \( A \times A \) has 9 elements among which are found \( \{1, 0\} \) and \( \{0, 1\} \). Find the set \( A \) and the remaining elements of \( A \times A \)
We know that if \( n(A) = p \) and \( n(B) = q \), then \( n(A \times B) = n(A) \times n(B) = pq \).

\[ n(A \times A) = n(A) \times n(A) \]

It is given that \( n(A \times A) = 9 \).

\[ n(A) \times n(A) = 9 \]

\[ n(A) = 3 \]

The ordered pairs \((-1, 0)\) and \((0, 1)\) are two of the elements of \( A \times A \).

Now, \( A \times A = \{(a, b) : a \in A\} \). Therefore \(-1, 0\) and \(1\) are elements of \( A \).

Since \( n(A) = 3 \), so set \( A = \{-1, 0, 1\} \).

The remaining elements of set \( A \times A \) are \((1, -1), (-1, 1), (0, 0), (1, 0), (0, -1), (0, 1)\) and \((1, 1)\).

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**#417893**

**Topic: Relations**

Let \( A = \{1, 2, 3, \ldots, 14\} \). Define a relation \( R \) from \( A \) to \( A \) by \( R = \{(x, y) : 3x - y = 0 \text{ where } x, y \in A\} \). Write down its domain, co-domain and range.

**Solution**

The relation \( R \) from \( A \) to \( A \) is given as

\[ R = \{(x, y) : 3x - y = 0 \text{ where } x, y \in A\} \]

i.e., \( R = \{(x, y) : 3x = y, x, y \in A\} \)

\[ R = \{(1, 3), (2, 6), (3, 9), (4, 12)\} \]

The domain of \( R \) is the set of all first elements of the ordered pairs in the relation.

\[ \text{Domain of } R = \{1, 2, 3, 4\} \]

The whole set \( A \) is the co-domain of the relation \( R \).

\[ \text{Codomain of } R = A = \{1, 2, 3, \ldots, 14\} \]

The range of \( R \) is the set of all second elements of the ordered pairs in the relation.

\[ \text{Range of } R = \{3, 6, 9, 12\} \]

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**#417896**

**Topic: Relations**

Define a relation \( R \) on the set \( N \) of natural numbers by \( R = \{(x, y) : y = x + 5, x \text{ is a natural number less than } 4, x, y \in N\} \). Depict this relationship using roster form. Write down the domain and the range.

**Solution**

Given definition of \( R \) is

\[ R = \{(x, y) : y = x + 5, x \text{ is a natural number less than } 4, x, y \in N\} \]

The natural numbers less than 4 are 1, 2 and 3.

\[ R = \{(1, 6), (2, 7), (3, 8)\} \]

The domain of \( R \) is the set of all first elements of the ordered pairs in the relation.

\[ \text{Domain of } R = \{1, 2, 3\} \]

The range of \( R \) is the set of all second elements of the ordered pairs in the relation.

\[ \text{Range of } R = \{6, 7, 8\} \]

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**#417898**

**Topic: Relations**

\( A = \{1, 2, 3, 5\} \) and \( B = \{4, 6, 9\} \). Define a relation \( R \) from \( A \) to \( B \) by \( R = \{(x, y) : \text{the difference between } x \text{ and } y \text{ is odd } x, y \in A, y \in B\} \). Write \( R \) in roster form.

**Solution**

\( A = \{1, 2, 3, 5\} \) and \( B = \{4, 6, 9\} \)

\[ R = \{(x, y) : \text{the difference between } x \text{ and } y \text{ is odd } x, y \in A, y \in B\} \]

\[ R = \{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\} \]

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**#417901**

**Topic: Relations**

\( A = \{1, 2, 3, 5\} \) and \( B = \{4, 6, 9\} \). Define a relation \( R \) from \( A \) to \( B \) by \( R = \{(x, y) : \text{the difference between } x \text{ and } y \text{ is odd } x, y \in A, y \in B\} \). Write \( R \) in roster form.
Let $A = \{1, 2, 3, 4, 6\}$ and $R$ be the relation on $A$ defined by $(a, b) \in R$ if $a, b \in A, b$ is exactly divisible by $a$.

(i) Write $R$ in roster form
(ii) Find the domain of $R$
(iii) Find the range of $R$

Solution

$A = \{1, 2, 3, 4, 6\}$

$R = \{(a, b): a, b \in A, b \text{ is exactly divisible by } a\}$

(i) $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (6, 6)\}$
(ii) Domain of $R = \{1, 2, 3, 4, 6\}$
(iii) Range of $R = \{1, 2, 3, 4, 6\}$

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#417902

**Topic:** Relations

Determine the domain and range of the relation $R$ defined by $R = \{(x, x + 5): x \in \{0, 1, 2, 3, 4, 5\}\}$

Solution

$R = \{(x, x + 5): x \in \{0, 1, 2, 3, 4, 5\}\}$

$\Rightarrow R = \{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$

$\therefore$ Domain of $R = \{0, 1, 2, 3, 4, 5\}$

Range of $R = \{5, 6, 7, 8, 9, 10\}$

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#417903

**Topic:** Relations

Write the relation $R = \{(x, x^2): x \text{ is a prime number less than 10}\}$ in roster form.

Solution

$R = \{(x, x^2): x \text{ is a prime number less than 10}\}$

The prime numbers less than 10 are 2, 3, 5 and 7.

$\therefore R = \{(2, 4), (3, 9), (5, 25), (7, 49)\}$

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#417904

**Topic:** Relations

Let $A = \{x, y, z\}$ and $B = \{1, 2\}$. Find the number of relations form $A$ to $B$.

Solution

It is given that $A = \{x, y, z\}$ and $B = \{1, 2\}$.

$\therefore A \times B = \{(x, 1), (x, 2), (y, 1), (y, 2), (z, 1), (z, 2)\}$

Since $n(A \times B) = 6$.

The number of subsets of $A \times B$ is $2^6$.

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#417905

**Topic:** Functions

Which of the following relations are functions? Give reasons.

If it is a function determine its domain and range.

(i) $\{(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)\}$

(ii) $\{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\}$

(iii) $\{(1, 3), (1, 5), (2, 5)\}$

Solution

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(i) \(\{(2, 1), (5, 1), (8, 1), (1, 1), (14, 1), (17, 1)\}\)
It is a function as every input has a single output.
So, 2, 5, 8, 11, 14 and 17 are the elements of the domain of the given relation.
Here domain = \{2, 5, 8, 11, 14, 17\} and range = \{1\}

(ii) \(\{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\}\)
It is a function as every input has a single output.
So, 2, 4, 6, 8, 10, 12 and 14 are the elements of the domain of the given relation.
Here domain = \{2, 4, 6, 8, 10, 12, 14\} and range = \{1, 2, 3, 4, 5, 6, 7\}

(iii) \(\{(1, 3), (1, 5), (2, 5)\}\)
Since the element 1 corresponds to two different images i.e., 3 and 5. So, this relation is not a function.

#417906

**Topic:** Functions

Find the domain and range of the following real function:

(i) \(f(x) = -\mid x\mid\)
(ii) \(f(x) = \sqrt{9-x^2}\)

**Solution**

(i) \(f(x) = -\mid x\mid, x \in \mathbb{R}\)
We know that \(\mid x\mid \begin{cases} 
- x, x \geq 0 \\
+ x, x < 0
\end{cases}\)
\(\therefore f(x) = -\mid x\mid \begin{cases} 
- x, x \geq 0 \\
+ x, x < 0
\end{cases}\)

Since, \(f(x)\) is defined for \(x \in \mathbb{R}\)
Domain of \(f\) is \(\mathbb{R}\)
It can be observed that the range of \(f(x) = -\mid x\mid\) is all real numbers except positive real numbers
\(\therefore\) The range of \(f\) is \([-\infty, 0]\)

(ii) \(f(x) = \sqrt{9-x^2}\)
For this function to be defined,
\(9-x^2 \geq 0\)
\(\Rightarrow -3 \leq x \leq 3\)
For any value of \(x\) such that \(-3 \leq x \leq 3\) the value of \(f(x)\) will lie between 0 and 3
\(\therefore\) The range of \(f\) is \([0, 3]\)

#417907

**Topic:** Functions

A function \(r\) is defined by \(r(x) = 2x - 5\). Write down the values of

(i) \(r(0)\)
(ii) \(r(7)\)
(iii) \(r(-3)\)

**Solution**

\(r\) is given by \(r(x) = 2x - 5\)
Then, we have
(i) \(r(0) = 2(0) - 5 = -5\)
(ii) \(r(7) = 2(7) - 5 = 9\)
(iii) \(r(-3) = 2(-3) - 5 = -11\)
#417908

**Topic:** Functions

The function \( f(x) \) which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by \( f(C) = \frac{9C}{5} + 32^\circ \)

Find:

(i) \( 60^\circ \)

(ii) \( 528^\circ \)

(iii) \( 10^\circ \)

(iv) The value of \( C \) when \( f(C) = 212^\circ F \)

**Solution**

The given function is

\[ f(C) = \frac{9C}{5} + 32^\circ \]

(i) \( 60^\circ = \frac{9 \times 0}{5} + 32^\circ = 0 + 32^\circ = 32^\circ F \)

(ii) \( 528^\circ = \frac{9 \times 28^\circ}{5} + 32^\circ = \frac{252^\circ}{5} + 32^\circ = 50.4^\circ F \)

(iii) \( 10^\circ = \frac{9 \times (-10^\circ)}{5} + 32^\circ = \frac{-90^\circ}{5} + 32^\circ = -18^\circ F \)

(iv) It is given that \( f(C) = 212^\circ F \)

\[ \frac{9C}{5} + 32^\circ = 212^\circ F \]

\[ 9C = 180^\circ \times 5 \]

\[ C = \frac{900^\circ}{9} = 100^\circ \]

Thus the value of Celsius temperature is \( 100^\circ \) when Fahrenheit temperature is \( 212^\circ \).

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#417910

**Topic:** Functions

Find the range of each of the following functions

(i) \( f(x) = 2 - 3x \), \( x \in \mathbb{R}, x > 0 \)

(ii) \( g(x) = x^2 + 2 \), \( x \) is a real number

(iii) \( h(x) = x \), \( x \) is a real number

**Solution**
(i) Given $x > 0$
\[ \Rightarrow 3x > 0 \]
\[ \Rightarrow -3x < 0 \]
\[ \Rightarrow 2 - 3x < 2 \]
\[ \Rightarrow \alpha(x) < 2 \]
\[ \therefore \text{Range of } f = (-\infty, 2) \]

(ii) Since, for any real number $x$, $x^2$ is a real number
\[ \Rightarrow x^2 + 2 \geq 0 + 2 \]
\[ \Rightarrow x^2 + 2 \geq 2 \]
\[ \Rightarrow \beta(x) \geq 2 \]
\[ \therefore \text{Range of } f = [2, \infty) \]

(iii) $\gamma(x) = x$, $x$ is a real number
It is clear that the range of $\gamma$ is the set of all real numbers
\[ \therefore \text{Range of } f = \mathbb{R} \]

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**#417913**

**Topic:** Functions

The relation $f$ is defined by $f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 < x \leq 10 \end{cases}$

The relation $g$ is defined by $g(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 3x, & 2 < x \leq 10 \end{cases}$

Show that $f$ is a function and $g$ is not a function.

**Solution**

The relation $f$ is defined as $f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 < x \leq 10 \end{cases}$

It is observed that for
\[ 0 \leq x \leq 3, \ f(x) = x^2 \]
\[ 3 \leq x \leq 10, \ f(x) = 3x \]

Also at $x = 3, \ f(3) = 3^2 = 9$, or $f(3) = 3 \times 3 = 9$

i.e., at $x = 3, \ f(x) = 9$

Therefore for $0 \leq x \leq 10$, the images of $f(x)$ are unique.

Thus the given relation is a function.

The relation $g$ is defined as $g(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 3x, & 2 < x \leq 10 \end{cases}$

It can be observed that for $x = 2$, $g(x) = 2^2 = 4$ and $g(x) = 3 \times 2 = 6$

Hence element 2 of the domain of the relation $g$ corresponds to two different images i.e., 4 and 6.

Hence, this relation is not a function.

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**#417914**

**Topic:** Functions

If $f(x) = x^2$ find $\frac{f(10) - f(5)}{(10 - 5)}$

**Solution**

Given, $f(x) = x^2$

\[ \frac{f(10) - f(5)}{(10 - 5)} = \frac{10^2 - 5^2}{5} = \frac{101 - 25}{5} = \frac{76}{5} = 15.2 \]

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**#417915**

**Topic:** Functions
Find the domain of the function \( f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12} \)

Solution

The given function is, \( f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12} \)

\[ f(x) = \frac{x^2 + 2x + 1}{(x - 6)(x - 2)} \]

It can be seen that function \( f \) is defined for all real numbers except at \( x = 6 \) and \( x = 2 \)

Hence the domain of \( f \) is \( \mathbb{R} - \{2, 6\} \)

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Find the domain and range of the real function \( f \) defined by \( f(x) = \sqrt{x-1} \)

Solution

The given real function is \( f(x) = \sqrt{x-1} \)

It can be seen that \( \sqrt{x-1} \) is defined for \( (x-1) \geq 0 \)

i.e., \( f(x) = \sqrt{x-1} \) is defined for \( x \geq 1 \)

Therefore the domain of \( f \) is the set of all real numbers greater than or equal to \( 1 \) i.e.,

the domain of \( f \) is \( [1, \infty) \)

As \( x \geq 1 \)

\[ (x-1) \geq 0 \]

\[ \sqrt{x-1} \geq 0 \]

\( f(x) \geq 0 \)

Therefore the range of \( f \) is the set of all real numbers greater than or equal to \( 0 \)

i.e., the range of \( f \) is \( (0, \infty) \)

---

Find the domain and range of the real function \( f \) defined by \( f(x) = |x-1| \)

Solution

The given real function is \( f(x) = |x-1| \)

It is clear that \( |x-1| \) is defined for all real numbers

i.e., \( f \) is defined for all real numbers

Thus for \( x \in \mathbb{R} \), \( |x-1| \) assumes all non-negative real numbers.

Hence the range of \( f \) is the set of all non-negative real numbers \( [0, \infty) \)

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Let \( f : \mathbb{R} \rightarrow \mathbb{R} : x \mapsto \frac{x^2}{1 + x^2} \) be a function from \( \mathbb{R} \) into \( \mathbb{R} \) Determine the range of \( f \)

Solution
Let \( \frac{x^2}{y} = \frac{1}{1+y} \)

\[ \Rightarrow y + y^2 = x^2 \]
\[ \Rightarrow y = x^2(1-y) \]
\[ \Rightarrow \frac{y}{x^2} = \frac{1}{1+y} \]
\[ \Rightarrow x = \sqrt{\frac{y}{1-y}} \]

Since, \( x \) is real

\[ \frac{y}{1-y} \geq 0 \]
\[ \Rightarrow |y| \geq 0 \]
\[ \Rightarrow y(1-y) \geq 0 \]
\[ \Rightarrow y(1-y) = 0 \text{ and } (1-y)^2 > 0 \]
\[ \Rightarrow 0 \leq y \leq 1 \text{ and } y > -1 \]
\[ \Rightarrow 0 \leq y < 1 \]

Hence, \( 0 \leq y < 1 \)

Range of \( f \) is \( [0, 1) \)

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**#417921**

**Topic:** Algebra of Real Functions

Let \( f, g : \mathbb{R} \rightarrow \mathbb{R} \) be defined respectively by \( f(x) = x + 1, g(x) = 2x - 3 \). Find \( f + g, f - g \), and \( \frac{f}{g} \).

**Solution**

\( f, g : \mathbb{R} \rightarrow \mathbb{R} \) is defined as

\[ f(x) = x + 1 \]
\[ g(x) = 2x - 3 \]

Now, \( (f+g)(x) = f(x) + g(x) = (x + 1) + (2x - 3) = 3x - 2 \)
\[ \therefore (f+g)(x) = 3x - 2 \]

Now, \( (f-g)(x) = f(x) - g(x) = (x + 1) - (2x - 3) = x + 1 - 2x + 3 = -x + 4 \)
\[ \therefore (f-g)(x) = -x + 4 \]

\[ \left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)} = \frac{x+1}{2x-3} \neq 0, x \in \mathbb{R} \]
\[ \therefore \left( \frac{f}{g} \right)(x) = \frac{x+1}{2x-3} \neq 0 \text{ or } 2x \neq 3 \]
\[ \therefore \left( \frac{f}{g} \right)(x) = \frac{x+1}{2x-3} \neq \frac{3}{2} \]

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**#417922**

**Topic:** Functions

Let \( f : [1, 3], \{0, -3\} \rightarrow [2, 3], \{0, -3\} \) be a function from \( [1, 3] \) defined by \( \xi(x) = ax + b \) for some integers \( a, b \). Determine \( a, b \).

**Solution**
\[ f = (1, 2, 3, 0, -1, -2) \]
\[ g = \{a \in \mathbb{R} : a = \sqrt{a^2} \} \]
\[ (a, b) \in f \]
\[ (0, 1) \in f \Rightarrow f(0) = 1 \]
\[ a + b = 1 \Rightarrow a = 1 \]
\[ (0, -1) \in f \Rightarrow f(0) = -1 \]
\[ a + b = -1 \Rightarrow b = -1 \]

On substituting \( b = -1 \) in eqn (i), we get
\[ a + (-1) = 1 \]
\[ a = 1 + 1 = 2 \]

Thus the respective values of \( a \) and \( b \) are 2 and -1

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**#417923**

**Topic**: Relations

Let \( R \) be a relation from \( N \) to \( N \) defined by \( R = \{(a, b) : a, b \in N \text{ and } a = b^2\} \). Are the following true?

(i) \( (a, b) \in R \) for all \( a \in N \)

(ii) \( (a, b) \in R \) implies \( (b, a) \in R \)

(iii) \( (a, b) \in R, (b, c) \in R \) implies \( (a, c) \in R \)

Justify your answer in each case

**Solution**

\[ R = \{(a, b) : a, b \in N \text{ and } a = b^2\} \]

(i) It can be seen that \( 2 \in N \) however, \( 2 \times 2^2 = 4 \)

Therefore the statement \( (a, a) \in R \) for all \( a \in N \) is not true

(ii) It can be seen that \( (9, 3) \in N \) because \( 9, 3 \in N \) and \( 9 = 3^2 \)

Now \( 3 \times 3^2 = 27 \)

\[ \therefore (3, 3) \notin N \]

Therefore the statement \( (a, b) \in R \) implies \( (b, a) \in R \) is not true.

(iii) It can be seen that \( (9, 3) \in R, (16, 4) \in R \) because \( 9, 3, 16, 4 \in N \) and \( 9 = 3^2 \) and \( 16 = 4^2 \)

Now \( 9 \times 4^2 = 144 \)

\[ \therefore (9, 4) \notin N \]

Therefore the statement \( (a, b) \in R, (b, c) \in R \) implies \( (a, c) \in R \) is not true

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**#418034**

**Topic**: Functions

Let \( A = \{1, 2, 3, 4\} \) and \( B = \{1, 5, 9, 11, 15, 16\} \) and \( f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\} \) Are the following true?

(i) \( f \) is a relation from \( A \) to \( B \)

(ii) \( f \) is a function from \( A \) to \( B \)

Justify your answer in each case

**Solution**
\[ A = \{1, 2, 3, 4\} \text{ and } B = \{1, 5, 9, 11, 15, 16\} \]

\[ A \times B = \{(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), (2, 11), (2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5), (4, 9), (4, 11), (4, 15), (4, 16)\} \]

It is given that \( r = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\} \)

(i) A relation from a non-empty set \( A \) to a non-empty set \( B \) is a subset of the Cartesian product \( A \times B \)

It is observed that \( r \) is a subset of \( A \times B \)

Thus \( r \) is a relation from \( A \) to \( B \)

(ii) Since the element 2 corresponds to two different images i.e., 9 and 11. So, relation \( r \) is not a function.

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**#418054**

**Topic:** Functions

Let \( r \) be the subset of \( \mathbb{Z} \times \mathbb{Z} \) defined by \( r = \{(a, b) : a, b \in \mathbb{Z}\} \). Is \( r \) a function from \( \mathbb{Z} \to \mathbb{Z} \)? Justify your answer.

**Solution**

The relation \( r \) is defined as

\[ r = \{(a, b) : a, b \in \mathbb{Z}\} \]

We know that a relation \( r \) from set \( A \) to set \( B \) is said to be a function if every element of set \( A \) has unique images in set \( B \).

Since \( 2, 6 \in \mathbb{Z} \),

\[ (2 \times 6, 2 \times 6) \in r \]

Again since, \(-2, -6 \in \mathbb{Z} \),

\[ (-2 \times -6, -2 \times (-6)) \in r \]

i.e., \((12, 8), (12, -8) \in r \)

It can be seen that the same first element i.e., 12 corresponds to two different images i.e., 8 and \(-8\).

Thus relation \( r \) is not a function.

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**#418064**

**Topic:** Functions

Let \( A = \{9, 10, 11, 12, 13\} \) and let \( r : A \to \mathbb{N} \) be defined by \( r(n) = \) the highest prime factor of \( n \). Find the range of \( r \).

**Solution**

\[ A = \{9, 10, 11, 12, 13\} \]

\( r : A \to \mathbb{N} \) is defined as

\[ r(n) = \text{The highest prime factor of } n \]

Prime factor of 9 = 3

Prime factors of 10 = 2, 5

Prime factors of 11 = 11

Prime factors of 12 = 2, 3

Prime factors of 13 = 13

\[ r(9) = \text{The highest prime factor of 9 = 3} \]

\[ r(10) = \text{The highest prime factor of 10 = 5} \]

\[ r(11) = \text{The highest prime factor of 11 = 11} \]

\[ r(12) = \text{The highest prime factor of 12 = 3} \]

\[ r(13) = \text{The highest prime factor of 13 = 13} \]

Thus range of \( r \) is the set of all \( n \) such that \( n \in A \)

\[ \text{Is } s = \{3, 5, 11, 13\} \]
How many elements has $P(A)$, if $A = \emptyset$?

**Solution**

We know that if $A$ is a set with $m$ elements i.e., $n(A) = m$, then $n(P(A)) = 2^m$.

If $A = \emptyset$, then $n(A) = 0$

$\therefore n(P(A)) = 2^0 = 1$

Hence, $P(A)$ has one element.

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#419705

**Topic:** Functions

Assume that $P(A) = P(B)$. Show that $A = B$

**Solution**

Let $P(A) = P(B)$

To show: $A = B$

Let $x \in A$

$A \in P(A) = P(B)$

$\therefore x \in C$ for some $C \in P(B)$

Now, $C \subseteq B$

$\therefore x \in B$

But $x$ is an arbitrary element in $A$

$\therefore A \subseteq B$  ——(1)

Now, let $y \in B$

$B \in P(B) = P(A)$

$\Rightarrow y \in D$ for some $D \in P(A)$

$D \subseteq A$

$\Rightarrow y \in A$

But $y$ is an arbitrary element in $B$.

Hence, $B \subseteq A$  ——(2)

From (1) and (2), we get

$A = B$

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#419708

**Topic:** Functions

Is it true that for any sets $A$ and $B$, $P(A) \cup P(B) = P(A \cup B)$? Justify your answer

**Solution**

False

Let $A = \{0, 1\}$ and $B = \{1, 2\}$

$\therefore A \cup B = \{0, 1, 2\}$

$P(A) = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\}$

$P(B) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$

$P(A) \cup P(B) = \{\emptyset, \{0\}, \{1\}, \{0, 1\}, \{2\}, \{1, 2\}\}$

$P(A \cup B) = \{\emptyset, \{0\}, \{1\}, \{0, 1\}, \{2\}, \{1, 2\}\}$

$\therefore P(A) \cup P(B) \neq P(A \cup B)$

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#446760

**Topic:** Relations

Let $R$ be the relation on $Z$ defined by $R = \{(a, b) : a, b \in Z, a - b$ is an integer$\}$. Find the domain and range of $R$.

**Solution**
Given $R = \{(a, b); a, b \in Z, a - b \text{ is an integer}\}$

$a \in Z$ so, Domain of $R$ is $Z$.

$R$ is such that $a - b \in Z$ and we have $a \in Z$ so $b \in Z$.

So, Range of $R$ is $Z$.

- Domain of $R = Z$ and Range of $R = Z$.

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**#458289**

*Topic: Relations*

![Diagram](image)

The figure shows a relationship between the sets $P$ and $Q$. Write this relation in:

(i) In set-builder form

(ii) Roster form

**Solution**

The relation mentioned in the figure shows, $P$ as domain and $Q$ as range.

Let the relation be $R$

In roster form $R = \{(5, 3), (6, 4), (7, 5)\}$

In set builder form $R = \{(x, y); x \in P, y \in Q, y = x - 2\}$

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**#459565**

*Topic: Special Functions*

Find the maximum and minimum values, if any, of the following function given by:

$f(x) = |x + 2| - 1$

**Solution**

$f(x) = |x + 2| - 1$

Here, $|x + 2|$ is always greater than $0$ (property of mod).

$|x + 2| \geq 0$

$\Rightarrow f(x) = |x + 2| - 1 \geq -1$

$\Rightarrow$ function has a minimum value of $-1$ and this happens when $|x + 2| = 0$

i.e. $x = -2$

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**#459566**

*Topic: Special Functions*

Find the maximum and minimum values, if any, of the following function given by:

$g(x) = -|x + 1| + 3$

**Solution**

$g(x) = -|x + 1| + 3$

As we know from property of mod that $|x + 1| \geq 0$

$\Rightarrow -|x + 1| \leq 0$ (When an inequality is multiplied from $-1$ then it get inverted)

$\Rightarrow -|x + 1| + 3 \leq 3$

$\Rightarrow g(x) \leq 3$

This shows that $g(x)$ will attain maximum value of $3$ and that to when $-|x + 1| = 0$

i.e. $x = -1$

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