#463677

Factorise the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$

(ii) $4y^2 - 4y + 1$

(iii) $x^2 - \frac{y^2}{100}$

Solution

(i) $9x^2 + 6xy + y^2 = (3x)^2 + 2(3x)(y) + (y)^2 = (3x + y)^2$

(ii) $4y^2 - 4y + 1 = (2y)^2 - 2(2y)(1) + 1 = (2y - 1)^2$

(iii) $x^2 - \frac{y^2}{100} = (x)^2 - \left(\frac{y}{10}\right)^2 = \left(x - \frac{y}{10}\right)\left(x + \frac{y}{10}\right)$

#463695

Factorise:

$27x^3 + y^3 + z^3 - 9xyz$

Solution

$27x^3 + y^3 + z^3 - 9xyz$

$= (3x)^3 + (y)^3 + z^3 - 3(3x)(y)(z)$

$= (3x + y + z)\left\{(3x)^2 + (y)^2 + (z)^2 - (3x)(y) - (y)(z) - (z)(3x)\right\}$

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$

#463696

Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z) + (z - x)^2]$

Solution

$x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= \frac{1}{2}(x + y + z)(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx)$

$= \frac{1}{2}\left\{(x - y)^2 + (y - z)^2 + (z - x)^2\right\}$

#463697

If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

Solution

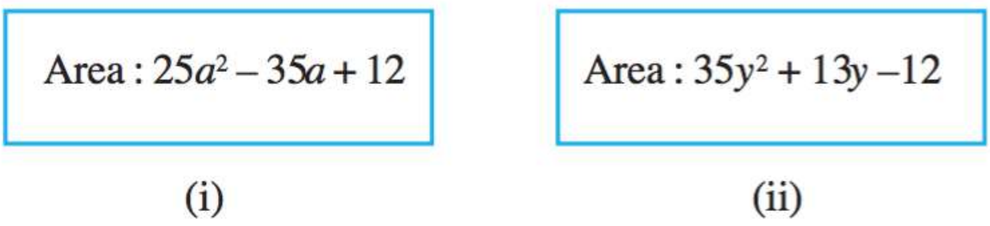
$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (0)(x^2 + y^2 + z^2 - xy - yz - zx)$ (Given $x + y + z = 0$)

$\therefore x^3 + y^3 + z^3 - 3xyz = 0$

$\therefore x^3 + y^3 + z^3 = 3xyz$

#463699



Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

Solution

(i) $25a^2 - 35a + 12$

$= 25a^2 - 20a + 15a + 12$

$= 5a(5a - 4) - 3(5a - 4) = (5a - 4)(5a - 3)$

The possible values of length $= (5a - 3)$ and

breadth $= (5a - 4)$.

(ii) $35y^2 + 13y - 12$

$= 25y^2 + 28y - 15y - 12$

$= 7y(5y + 4) - 3(5y + 4) = (5y + 4)(7y - 3)$

The possible values of length $= (7y - 3)$

and breadth $= (5y + 4)$.

#463700

Volume : $3x^2 - 12x$	Volume : $12ky^2 + 8ky - 20k$
(i)	(ii)

What are the possible expressions for the dimension of the cuboids whose volumes are given as in image?

Solution

(i)

$3x^2 - 12x = 3x(x - 4)$

$= 3 \times x \times (x - 4)$,

The possible values of dimensions are 3, x and $x - 4$.

(ii)

$12ky^2 + 8ky - 20k$

$= 4k(3y^2 + 2y - 5)$

$= 4k(3y^2 + 5y - 3y + 5)$

$= 4k\{y(3y + 5) - 1(3y + 5)\}$

$= 4k(3y + 5)(y - 1)$

The possible values of dimensions are $4k$, $(3y + 5)$ and $(y - 1)$.

#463767

Find the common factors of the given terms.

(i) $12x, 36$

(ii) $2y, 22xy$

(iii) $14pq, 28p^2q^2$

(iv) $2x, 3x^2, 4$

(v) $6abc, 24ab^2, 12a^2b$

(vi) $16x^3, -4x^2, 32x$

(vii) $10pq, 20qr, 30rp$

(viii) $3x^2y^3, 10x^3y^2, 6x^2y^2z$

Solution

(i) $12x = 2 \times 2 \times 3 \times x$

$36 = 2 \times 2 \times 3 \times 3$

Common factor $= 2 \times 3 = 6$

(ii) $2y = 2 \times y$

$22y = 2 \times 11 \times y$

Common factor $= 2$

(iii) $14pq = 2 \times 7 \times p \times q$

$28p^2q^2 = 2 \times 2 \times 7 \times p \times p \times q \times q$

Common factor $= 14pq$

(iv) $2x = 2 \times x$

$3x^2 = 3 \times x \times x$

$4 = 2 \times 2$

Common factor $= 1$

(v) $6abc = 6 \times a \times b \times c$

$24ab^2 = 2 \times 2 \times 2 \times 3 \times a \times b \times b$

$12a^2b = 2 \times 2 \times 3 \times a \times a \times b$

Common factor $= 6ab$

(vi) $16x^3 = 2 \times 2 \times 2 \times 2 \times x \times x \times x$

$-4x^2 = -2 \times 2 \times x \times x$

$32x = 2 \times 2 \times 2 \times 2 \times 2 \times x$

Common factor $= 2x$

(vii) $10pq = 2 \times 5 \times p \times q$

$20qr = 2 \times 2 \times 5 \times q \times r$

$30rp = 2 \times 3 \times 5$

Common factor $= 2 \times 5 = 10$

(viii) $3x^2y^3 = 3 \times x \times x \times y \times y \times y$

$10x^3y^2 = 2 \times 5 \times x \times x \times x \times y \times y$

$6x^2y^2z = 2 \times 3 \times x \times x \times y \times y \times z$

Common factor = x^2y^2

#463768

Factorise the following expressions.

(i) $7x - 42$

(ii) $6p - 12q$

(iii) $7a^2 + 14a$

(iv) $$-16z +20z^2

(v) $20l^2m + 30alm$

(vi) $5x^2y - 15xy^2$

(vii)$10a^2 - 15b^2 + 20c^2$

(viii) $-4a^2 + 4ab - 4ca$

(ix) $x^2yz + xy^2z + xyz^2$

(x) $ax^2y + bxy^2 + cxyz$

Solution

(i) $7x - 42 = 7(x - 6)$

(ii) $6p - 12q = 6(p - 2q)$

(iii) $7a^2 + 14a = 7a(a + 2)$

(iv) $-16z + 20z^2 = 4z(5z^2 - 4)$

(v) $20l^2m + 30alm = 2lm(10l + 15a)$

(vi) $5x^2y - 15xy^2 = 5x(xy - 3y^2)$

(vii)$10a^2 - 15b^2 + 20c^2 = 5(2a^2 - 3b^2 + 4c^2)$

(viii) $-4a^2 + 4ab - 4ca = -4a(a - b + c)$

(ix) $x^2yz + xy^2z + xyz^2 = xyz(x + y + z)$

(x) $ax^2y + bxy^2 + cxyz = xyz(ax + by + cz)$

#463769

Factorise

(i) $x^2 + xy + 8x + 8y$

(ii) $15xy - 6x + 5y - 2$

(iii) $ax + by - ay - by$

(iv) $15pq + 15 + 9q + 25p$

(v) $z - 7 + 7xy - xyz$

Solution

(i) $x^2 + xy + 8x + 8y$

$= x(x + y) + 8(x + y)$

$= (x + 8)(x + y)$

(ii) $15xy - 6x + 5y - 2$

$= 3x(5y - 2) + 1(5y - 2)$

$= (3x + 1)(5y - 2)$

(iii) $ax + by - ay - by$

$= x(a + b) - y(a + b)$

$= (x - y)(a + b)$

(iv) $15pq + 15 + 9q + 25p$

$= 3q(5p + 3) + 5(5p + 3)$

$= (3q + 5)(5p + 3)$

(v) $z - 7 + 7xy - xyz$

$= z(1 - xy) - 7(1 - xy)$

$= (z - 7)(1 - xy)$

#463770

Factorise the following expressions.

(i) $a^2 + 8a + 16$

(ii) $p^2 - 10p + 25$

(iii) $25m^2 + 30m + 9$

(iv) $49y^2 + 84yz + 36z^2$

v) $4x^2 - 8x + 4$

(vi) $121b^2 - 88bc + 16c^2$

(vii) $(l + m)^2 - 4lm$

(viii) $a^4 + 2a^2b^2 + b^4$

Solution

(i) $a^2 + 8a + 16$

$= a(a + 4) + 4(a + 4)$

$= (a + 4)(a + 4)$

(ii) $p^2 - 10p + 25$

$= p(p - 5) - 5(p - 5)$

$= (p - 5)(p - 5)$

(iii) $25m^2 + 30m + 9$

$= 5m(5m + 3) + 3(5m + 3)$

$= (5m + 3)(5m + 3)$

(iv) $49y^2 + 84yz + 36z^2$

$= (7y + 6z)(7y + 6z)$

(v) $4x^2 - 8x + 4$

$= (4x - 4)(x - 1)$

$= 4(x - 1)(x - 1)$

(vi) $121b^2 - 88bc + 16c^2$

$= (11b - 4c)(11b - 4c)$

(vii) $(l + m)^2 - 4lm$

$= l^2 + m^2 + 2lm - 4lm$

$= l^2 + m^2 - 2lm$

$= (l - m)(l - m)$

(viii) $a^4 + 2a^2b^2 + b^4$

$= (a^2)^2 + 2a^2b^2 + (b^2)^2$

$= [a^2 + b^2]^2$

#463771

Factorise

(i) $4p^2 - 9q^2$

(ii) $63a^2 - 112b^2$

(iii) $49x^2 - 36$

(iv) $16x^5 - 144x^3$

(v) $(l + m)^2 - (l - m)^2$

(vi) $9x^2y^2 - 16$

(vii) $(x^2 - 2xy + y^2) - z^2$

(viii) $25a^2 - 4b^2 + 28bc - 49c^2$

Solution

(i) $4p^2 - 9q^2$

$= (2p)^2 - (3q)^2$

$= (2p + 3q)(2p - 3q)$

(ii) $63a^2 - 112b^2$

$= 7[(3a)^2 - (4b)^2]$

$= 7(3a + 4b)(3a - 4b)$

(iii) $49x^2 - 36$

$= [(7x)^2 - (6)^2]$

$= (7x - 6)(7x + 6)$

(iv) $16x^5 - 144x^3$

$= 16x^3[x^2 - 3^2]$

$= 16x^3(x - 3)(x + 3)$

(v) $(l + m)^2 - (l - m)^2$

$= 2m \times 2l = 4lm$

(vi) $9x^2y^2 - 16$

$= [(3xy)^2 - 4^2]$

$= (3xy - 4)(3xy + 4)$

(vii) $(x^2 - 2xy + y^2) - z^2$

$= [(x - y)^2 - z^2]$

$= (x - y + z)(x - y - z)$

(viii) $25a^2 - 4b^2 + 28bc - 49c^2$

$= (5a)^2 - [(2b - 7c)^2]$

$= (5a + 2b - 7c)(5a - 2b + 7c)$

#463848

Factorise the expression

(i) $ax^2 + bx$

(ii) $7p^2 + 21q^2$

(iii) $2x^3 + 2xy^2 + 2xz^2$

(iv) $am^2 + bm^2 + bn^2 + an^2$

(v) $(lm + l) + m + 1$

(vi) $y(y + z) + 9(y + z)$

(vii) $5y^2 - 20y - 8z + 2yz$

(viii) $10ab + 4a + 5b + 2$

(ix) $6xy - 4y + 6 - 9x$

Solution

(i) $ax^2 + bx$

$= x(ax + b)$

(ii) $7p^2 + 21q^2$

$= 7(p^2 + 3q^2)$

(iii) $2x^3 + 2xy^2 + 2xz^2$

$= 2x(x^2 + y^2 + z^2)$

(iv) $am^2 + bm^2 + bn^2 + an^2$

$= m^2(a + b) + n^2(a + b)$

$= (a + b)(m^2 + n^2)$

(v) $(lm + l) + m + 1$

$= (l + 1)(m + 1)$

(vi) $y(y + z) + 9(y + z)$

$= (y + z)(y + 9)$

(vii) $5y^2 - 20y - 8z + 2yz$

$= (y - 4)(5y + 2z)$

(viii) $10ab + 4a + 5b + 2$

$= (2a + 1)(5b + 2)$

(ix) $6xy - 4y + 6 - 9x = 3x(2y - 3) - 2(2y - 3)$

$= (2y - 3)(3x - 2)$

#463853

Factorise

(i) $a^4 - b^4$

(ii) $p^4 - 81$

(iii) $x^4 - (y + z)^4$

(iv) $x^4 - (x - z)^4$

(v) $a^4 - 2a^2b^2 + b^4$

Solution

(i) $a^4 - b^4 = (a - b)(a + b)(a^2 + b^2)$

(ii) $p^4 - 81 = (p^2 - 9)(p^2 + 9) = (p - 3)(p + 3)(p^2 + 9)$

(iii) $x^4 - (y + z)^4 = [x^2 - (y + z)^2][x^2 + (y + z)^2]$

$= (x - y - z)(x + y + z)[x^2 + (y + z)^2]$

(iv) $x^4 - (x - z)^4 = [(x^2)^2 - (x - z)^{2^2}]$

$= z(2x - z)(2x^2 - 2xz + z^2)$

(v) $a^4 - 2a^2b^2 + b^4 = a^2(a^2 - b^2) - b^2(a^2 - b^2)$

$= (a - b)^2(a + b)^2$

#463860

Factorise the following expression

(i) $p^2 + 6p + 8$

(ii) $q^2 - 10q + 21$

(iii) $p^2 + 6p - 16$

Solution

Factorise the following expression

(i) $p^2 + 6p + 8$

$= p(p + 2) + 4(p + 2)$

$= (p + 2)(p + 4)$

(ii) $q^2 - 10q + 21$

$= q(q - 7) - 3(q - 7)$

$= (q - 3)(q - 7)$

(iii) $p^2 + 6p - 16$

$= p(p + 8) - 2(p + 8)$

$= (p + 8)(p - 2)$