Limit of Continuity

Limit :

1. l is the limit of f(x) if $x \to a$, $f(x) \to l$
2. Symbolically written as

$$\lim_{x \to a} f(x) = l$$

Right hand limit and Left hand limit

1. Expected value of f(x) at x=c when approached from left side of a
2. Symbolically written as

$$\lim_{x \to a^-} f(x) = l$$

3. Expected value of f(x) at x=c when approched from right side of a.
4. Symbolically written as

$$\lim_{x \to a^+} f(x) = l$$

5. Limit exists if

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

6. Limit does not exist if

$$\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$$

Algebra of limits

For f(x) and g(x),

$\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, we have,

1. $\lim_{x \to a} f(x) \pm g(x) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

2. $\lim_{x \to a} (f(x) \times g(x)) = \lim_{x \to a} f(x) \times \lim_{x \to a} g(x)$

3. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$; $\lim_{x \to a} g(x) \neq 0$

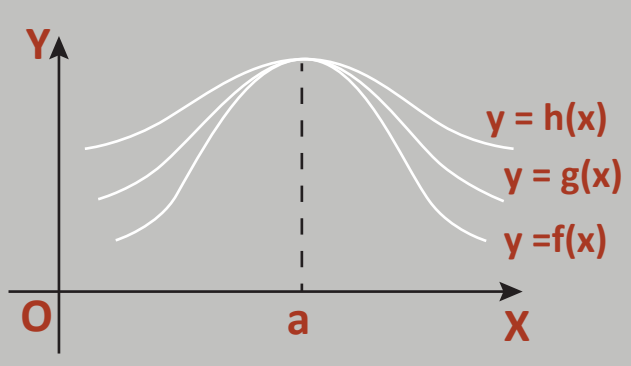
4. $\lim_{x \to a} (\lambda f(x)) = \lambda \lim_{x \to a} f(x)$

Sandwich Theorem

Let f(x) , g(x) and h(x) be real valued functions such that $f(x) \leq g(x) \leq h(x)$ for all x.
For a real number a

If $\lim_{x \to a} f(x) = l = \lim_{x \to a} h(x)$ then

$\lim_{x \to a} g(x) = l$



Important Limits

1. $\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{x}{\sin x} = 1$

2. $\lim_{x \to 0} \frac{\tan x}{x} = \lim_{x \to 0} \frac{x}{\tan x} = 1$

3. $\lim_{x \to 0} \frac{\log (1+x)}{x} = 1$

4. $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

5. $\lim_{x \to 0} \frac{a^x - 1}{x} = \log a$

6. $\lim_{x \to 0} \frac{(1 + x)^{1/x}}{x} = e$

7. $\lim_{x \to \infty} (1 + \frac{1}{x})^x = e$

8. $\lim_{x \to 0} \frac{x^n - a^n}{x - a} = n a^{n-1}$

9. $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

Continuity

1. A function f(x) is continuous at x=c, if limit exist and

$$\lim_{x \to c} f(x) = f(c)$$

2. A function f(x) is discontinuous at x=c, if

$$\lim_{x \to c} f(x) \neq f(c)$$

In this case, c is a point of discontinuity of f(x)

Algebra of continuous function

For two real valued function f(x) and g(x), continuous at x=c , we have

1. f(x) + g(x) continuous at x = c
2. f(x) - g(x) continuous at x = c
3. f(x) × g(x) continuous at x = c
4. $\frac{f(x)}{g(x)}$ is continuous at x = c , provided $g(c) \neq 0$