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Multiplication Of Matrices

- Matrix product AB is defined only when the number of columns of A is equal to number of rows of B
- For a matrix A of order i x j and matrix B of order j x k. Product AB results in matrix C of oder i x k.
- Each element of C is calculated as follow:

$$C_{ik} = \sum_j A_{ij} B_{jk}$$

Where C_{ik} is $(i, k)^{th}$ element of C
A_{ij} is $(i, j)^{th}$ element of A
B_{jk} is $(j, k)^{th}$ element of B
$\sum_j$ summation sign, which indicates that the a_{ij} b_{jk} terms should be summed over j.

Properties Of Matrix Multiplication

For three matrices A, B , C.

1. **Associative Law:** (AB)C = A(BC)
2. **Distributive Law:** A (B + C) = AB + AC
3. **Multiplicative Identity** : AI = IA = A

Transpose Of Matrix

- Matrix obtained by interchanging rows and columns of original matrix
- For matrix A, transpose of A is denoted by A^T or A'

Properties Of Transpose Matrix

1. $(A^T)^T = A$
2. $(KA^T)^T = KA^T$ where K is a scalar.
3. $(A + B)^T = A^T + B^T$
4. $(AB)^T = B^T A^T$
5. $(ABC)^T = C^T B^T A^T$

Symmetric Matrix

For a square matrix A, $A^T = A$

Skew-Symmetric Matrix

For a square matrix A, $A^T = -A$

Properties Of Symmetric Matrix And Skew-Symmetric Matrix

1. $A + A^T$ is a symmetric matrix.
2. $A - A^T$ is a skew-symmetric matrix.
3. $A = ½(A + A^T) + ½(A + A^T)$
   = ½(symmetric matrix) + ½(skew-symmetric matrix)

Invertible Matrices

- For square matrix A and B, if AB = BA = I then B is the inverse matrix of A.
- Inverse of matrix A is denoted by A^{-1}
- For a given square matrix there is one unique inverse matrix.
- For an inverse of A to exist, $|A| \neq 0$

Properties Of Inverse Matrix

1. $(A^T)^{-1} = (A^{-1})^T$
2. $(AB)^{-1} = B^{-1}A^{-1}$
3. $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$
4. $(A^{-1})^{-1} = A$