

Multiplication Of Matrices

- Matrix product AB is defined only when the number of columns of A is equal to number of rows of B
- For a matrix A of order $i \times j$ and matrix B of order $j \times k$. Product AB results in matrix C of order $i \times k$.
- Each element of C is calculated as follow:

$$C_{ik} = \sum_j A_{ij} B_{jk}$$

Where C_{ik} is $(i, k)^{\text{th}}$ element of C

A_{ij} is $(i, j)^{\text{th}}$ element of A

B_{jk} is $(j, k)^{\text{th}}$ element of B

\sum_j summation sign, which indicates that the a_{ij} b_{jk} terms should be summed over j .

Properties Of Matrix Multiplication

For three matrices A, B, C .

- 1 **Associative Law:** $(AB)C = A(BC)$
- 2 **Distributive Law:** $A(B + C) = AB + AC$
- 3 **Multiplicative Identity:** $AI = IA = A$

Transpose Of Matrix

- Matrix obtained by interchanging rows and columns of original matrix
- For matrix A , transpose of A is denoted by A^T or A'

Properties Of Transpose Matrix

- 1 $(A^T)^T = A$
- 2 $(KA^T)^T = KA^T$
where K is a scalar.
- 3 $(A + B)^T = A^T + B^T$
- 4 $(AB)^T = B^T A^T$
- 5 $(ABC)^T = C^T B^T A^T$

Symmetric Matrix

For a square matrix A , $A^T = A$

Skew-Symmetric Matrix

For a square matrix A , $A^T = -A$

Properties Of Symmetric Matrix And Skew-Symmetric Matrix

- 1 $A + A^T$ is a symmetric matrix.
- 2 $A - A^T$ is a skew-symmetric matrix.
- 3 $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$
 $= \frac{1}{2}(\text{symmetric matrix}) + \frac{1}{2}(\text{skew-symmetric matrix})$

Invertible Matrices

- For square matrix A and B , if $AB = BA = I$ then B is the inverse matrix of A .
- Inverse of matrix A is denoted by A^{-1}
- For a given square matrix there is one unique inverse matrix.
- For an inverse of A to exist, $|A| \neq 0$

Properties Of Inverse Matrix

- 1 $(A^T)^{-1} = (A^{-1})^T$
- 2 $(AB)^{-1} = B^{-1}A^{-1}$
- 3 $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$
- 4 $(A^{-1})^{-1} = A$