

Permutation And Combination

Fundamental Principle of Counting

For several mutually independent events $P_1, P_2, P_3, \dots, P_n$ which can occur in $n(P_1), n(P_2), n(P_3), \dots, n(P_n)$ ways.

Addition Rule

The no. of ways in which either P_1 or P_2 or P_3 or or P_n occurs is
$$n(E) = n(P_1) + n(P_2) + n(P_3) + \dots + n(P_n)$$

Product Rule

The no. of ways in which P_1 and P_2 and P_3 and and P_n occur is
$$n(E) = n(P_1) \times n(P_2) \times n(P_3) \times \dots \times n(P_n).$$

Factorial Notation

It is denoted by $n!$ and defined as

$$n! = n \times (n-1) \times (n-2) \times \dots \times 3 \times 2 \times 1$$

where n is a natural number

We also define $0! = 1$

Permutation

It is an arrangement, in a defined order, of objects taken some or all at a time.

Permutation when all objects are distinct

Permutation of n different objects taken r at a time, where $0 \leq r \leq n$,

without repetition is, ${}^n P_r = \frac{n!}{(n-r)!}$

and with repetition of objects is n^r .

Permutation when all objects are not distinct

Permutation of n objects where

P_1 objects are of one kind,

P_2 objects are of second kind,

.

.

P_k objects are of k^{th} kind and the rest if any, are of different kind is

$$= \frac{n!}{P_1! P_2! \dots P_k!}$$

Properties of ${}^n P_r$

1 ${}^n P_n = n!$

2 ${}^n P_1 = n$

3 ${}^n P_0 = 1$

4 ${}^n P_{n-1} = n!$

Combination

1 It is selection of r objects from n objects where order of selection does not matter. Here, $0 \leq r \leq n$.

2 It is denoted as (n, r) or ${}^n C_r$ calculated as

$${}^n C_r = \frac{n!}{r! (n-r)!}$$

$${}^n C_r = \frac{{}^n P_r}{r!}$$

Properties of ${}^n C_r$

1 ${}^n C_0 = {}^n C_n = 1$

2 ${}^n C_r = {}^n C_{n-r}$

3 If ${}^n C_r = {}^n C_p$ then either $r = p$ or $r + p = n$

4 ${}^n C_1 = n$