

# Integration - Part I

## Indefinite Integrals

- 1 Integration is the inverse process of differentiation

For  $\frac{d}{dx} g(x) = f(x)$ , we have,  $\int f(x) dx = g(x) + C$

- 2 These integrals are indefinite integrals, C is called the constant of integration  $n \neq 1$   
 3 Geometrically, it represents a collection of family of curves.

## Properties of Indefinite Integrals

1  $\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$

2  $\int k f(x) dx = k \int f(x) dx$   
when k is a constant

## Standard Integrals

1  $\int dx = x + c$

10  $\int \frac{dx}{1+x^2} = \cot^{-1} x + c$

2  $\int x^n dx = \frac{x^{n+1}}{n+1}$

11  $\int \frac{dx}{x \sqrt{x^2-1}} = \sec^{-1} x + c$

3  $\int \cos x dx = \sin x + c$

12  $\int \frac{dx}{x \sqrt{x^2-1}} = \operatorname{cosec}^{-1} x + c$

4  $\int \sin x dx = -\cos x + c$

13  $\int e^x dx = e^x + c$

5  $\int \sec^2 x dx = \tan x + c$

14  $\int a^x dx = \frac{a^x}{\log a} + c$

6  $\int \operatorname{cosec}^2 x dx = -\cot x + c$

15  $\int \frac{dx}{x} = \log |x| + c$

7  $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$

16  $\int \sec x \tan x dx = \sec x + c$

8  $\int \frac{dx}{\sqrt{1-x^2}} = \cos^{-1} x + c$

17  $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + c$

## Integration by Partial Fraction

1  $\frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$ ,  $a \neq b$

2  $\frac{px+q}{(x-a)^2} = \frac{A}{x-a} + \frac{B}{(x-a)^2}$

3  $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$

4  $\frac{px^2+qx+r}{(x-a)^2(x-b)} = \frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$

5  $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)} = \frac{A}{(x-a)} + \frac{Bx+C}{x^2+bx+c}$ ,

where  $x^2+bx+c$  cannot be factorized.

# Integration - Part II

## Integration by Substitution

In this variable is substituted in terms of other variable to get one of the standard integrals. Using this technique some more standard integrals are:

1  $\int \tan x dx = \log |\sec x| + c$

2  $\int \cot x dx = \log |\sin x| + c$

3  $\int \sec x dx = \log |\sec x + \tan x| + c$

4  $\int \operatorname{cosec} x dx = \log |\operatorname{cosec} x - \cot x| + c$

## Integral of Some Special Function

1  $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$

2  $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{x+a}{x-a} \right| + c$

3  $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + c$

4  $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log |x + \sqrt{x^2 - a^2}| + c$

5  $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \left( \frac{x}{a} \right) + c$

6  $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log |x + \sqrt{x^2 + a^2}| + c$

## Integration By Parts

- 1 For  $u = f(x)$  and  $v = g(x)$ , we have

$$\int u v dx = \int v dx - \int \left[ \frac{du}{dx} \int v dx \right] dx$$

- 2 The order of function is decided by the ILATE rule, where.



## Integrals of Special Type

1  $\int e^x [f(x) + f'(x)] dx = e^x f(x) + c$

2  $\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log \left| x + \sqrt{x^2 - a^2} \right| + c$

3  $\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log \left| x + \sqrt{x^2 + a^2} \right| + c$

4  $\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) + c$

# Integration - Part III

## Definite Integration

- 1 Integral of function with limits of integration it is denoted as

$I = \int_a^b f(x) dx$

where a = lower limit of integration,

b = upper limit of integration.

- 2 Definite integrals have a unique value.

3 If  $f = \int f(x) dx = g(x) + c$

then  $\int f_a^b(x) dx = g(b) - g(a)$

## Definite Integral as Limit of Sum

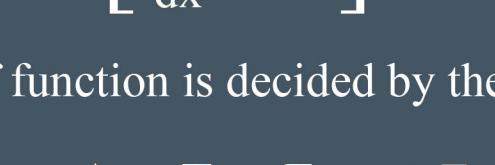
$${}_a^b f(x) dx = \lim_{n \rightarrow \infty} \frac{b-a}{n} \left[ f(a) + f(a+h) + \dots + f(a+(n-1)h) \right]$$

$$= \lim_{h \rightarrow 0} h \sum_{r=0}^{n-1} f(a+rh)$$

where  $h = \frac{b-a}{n}$

## Area function

- 1  $I = \int_a^b f(x) dx$  represents the area of region covered by  $f(x)$ , x - axis and lines  $x = a$  and  $x = b$ .



If  $x$  is a point in  $[a,b]$  then

$$I = \int_a^x f(x) dx = A(x)$$

this is known as area function as the value of  $A(x)$  depends on the value of  $x$

## Fundamental theorem of calculus

- 1 First Fundamental Theorem of Calculus

If  $f$  is a continuous function in  $[a,b]$  and  $A(x)$  is area function then

$$A'(x) = f(x), \text{ for all } x \in [a,b]$$

- 2 Second Fundamental Theorem of Calculus

If  $f$  is a continuous function in  $[a,b]$  and  $F$  is anti-derivative of  $f$  then

$$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a)$$

## Properties of Definite integrals

1  $\int_a^b f(x) dx = \int_a^b f(t) dt$

2  $\int_a^b f(x) dx = - \int_b^a f(x) dx$

3  $\int_a^a f(x) dx = 0$

4  $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$ ,

5  $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

6  $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

7  $\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a-x) dx$

8  $\int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx$  if  $f(2a-x) = f(x)$

$$= 0, \text{ if } f(2a-x) = -f(x)$$

9  $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$ , if  $f(x) = f(-x)$  i.e.  $f(x)$  is even function

$$= 0, \text{ if } f(x) = -f(-x) \text{ i.e. } f(x)$$

is an odd function