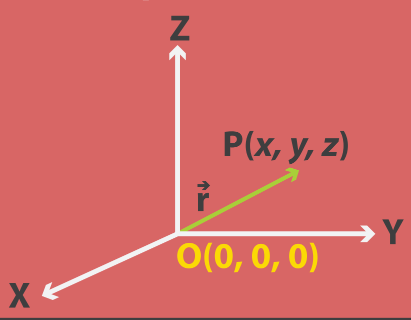
Vector Algebra - Part I

Vector It is a quantity which has both magnitude and direction.

Position Vector

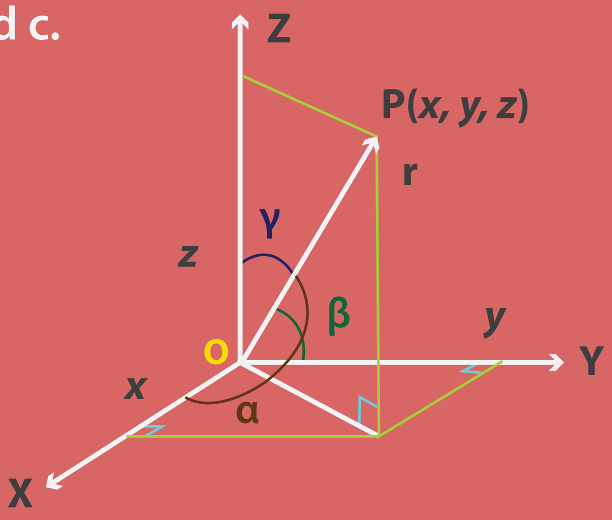
For a point P(x, y, z) in 3-D coordinate system, $\overline{OP}$ or $\bar{r}$ is the position vector with origin O as initial point.

Magnitude of $\overline{OP} = |\overline{OP}| = \sqrt{x^2 + y^2 + z^2}$



Direction Cosines and Direction Ratios

1. Position vector $\overline{OP}$ makes angle α, β and γ with x, y and z axes.
2. cos α, cos β and cos γ are the direction cosine of vector $\overline{OP}$ and denoted by l, m and n.
3. Direction cosines are unique for a given line
4. Number proportional to direction cosines are called as direction ratios, denoted by a, b and c.



5. We also have $l^2 + m^2 + n^2 = 1$.

Types Of Vectors

1. **Zero Vector** Vector with zero magnitude, denoted as $\bar{0}$
2. **Unit Vector** Vector whose magnitude is 1 unit. Unit vector along $\bar{a}$ is denoted as $\hat{a}$
3. **Cointial Vectors** Two or more vectors with same initial point
4. **Collinear Vectors** Two or more vectors lying on the same or parallel lines.
5. **Equal Vectors** Two or more vectors with same magnitude and direction.
6. **Negative Vectors** Vector with same magnitude but opposite direction as that of the given vector.

Addition Of Vectors

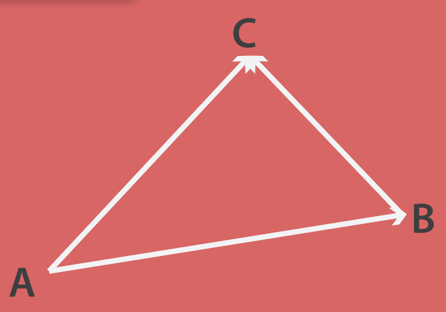
Triangle Law Of Addition

$\overline{AC} = \overline{AB} + \overline{BC}$

i.e $\overline{AB} + \overline{BC} - \overline{AC} = 0$

$\overline{AB} + \overline{BC} + \overline{CA} = 0$

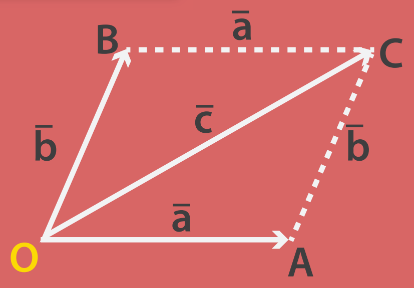
$\overline{AB} + \overline{BC} + \overline{CA} = \overline{AA}$



Parallelogram Law Of Addition

$\overline{OA} + \overline{OB} = \overline{OC}$

i.e $\bar{a} + \bar{b} = \bar{c}$



Properties Of Vector Addition

$\bar{a} + \bar{b} = \bar{b} + \bar{a}$; commutative property

$(\bar{a} + \bar{b}) + \bar{c} = \bar{a} + (\bar{b} + \bar{c})$; Associative property

Vector Algebra - Part II

Multiplication Of Scalar and Vector

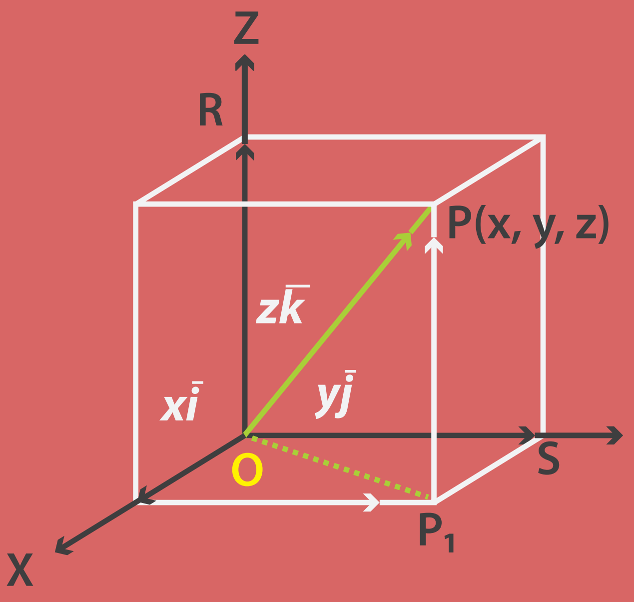
1. Multiplication of scalar λ with vector $\bar{a}$ is $\lambda\bar{a}$.
2. $\lambda\bar{a}$ is collinear to vector $\bar{a}$.
3. Direction of $\lambda\bar{a}$ depends upon λ.
   a. If λ is positive its direction is same as that of $\bar{a}$.
   b. If λ is negative its direction is opposite to that of $\bar{a}$.
4. Magnitude of $\lambda\,\bar{a} = |\lambda\,\bar{a}|$
   $= |\lambda|\,|\bar{a}|$

Components Of Vectors

For position vector OP of a point P(x, y, z), We have

$$\overline{OP} = x\bar{i} + y\bar{j} + z\bar{k}$$

Where $\bar{i}$, $\bar{j}$ and $\bar{k}$ are unit vectors along x, y and z - axes , and



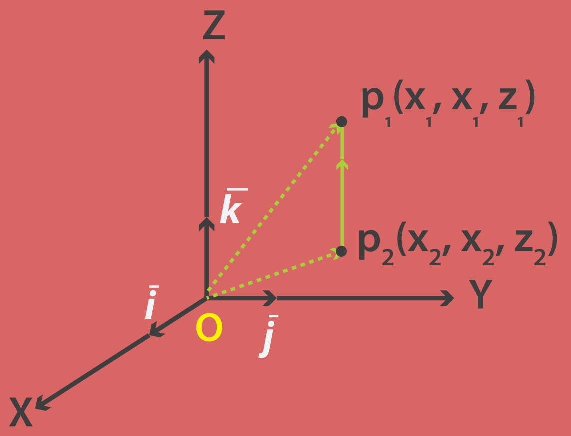
$x\bar{i}$, $y\bar{j}$ and $z\bar{k}$ are the components of vector OP along x , y and z - axes.

Vector Joining Two Points

$p_1(x_1, x_1, z_1)$ $p_2(x_2, x_2, z_2)$

$$\overline{p_1p_2} = (x_2 - x_1)\bar{i} + (y_2 - y_1)\bar{j} + (z_2 - z_1)\bar{k}$$

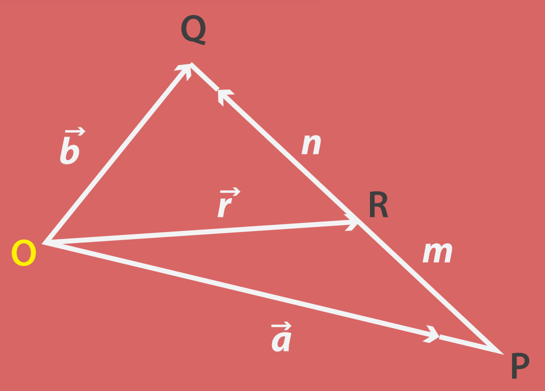
$$|\overline{p_1p_2}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



Section Formula

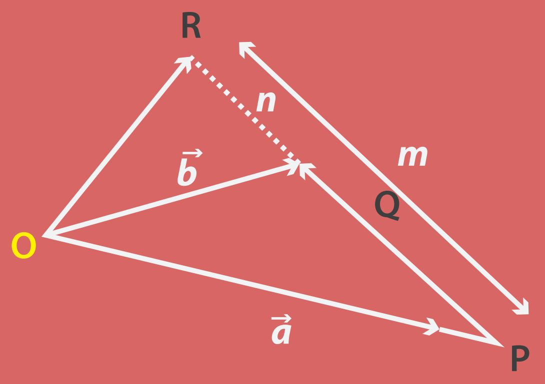
Internal Division

$$\bar{r} = \frac{m\bar{b} + n\bar{a}}{m + n}$$



External Division

$$\bar{r} = \frac{m\bar{b} - n\bar{a}}{m - n}$$



Midpoint Formula

$$\bar{r} = \frac{\bar{a} + \bar{b}}{2}$$

Vector Algebra - Part III

Scalar Product

For vectors $\bar{a}$ and $\bar{b}$. It is denoted as

$$\bar{a}.\bar{b} = |\bar{a}|\,|\bar{b}| \cos\theta$$

Where θ is angle between a and b , $0 \leq \theta \leq \pi$

Properties Of Scalar Product

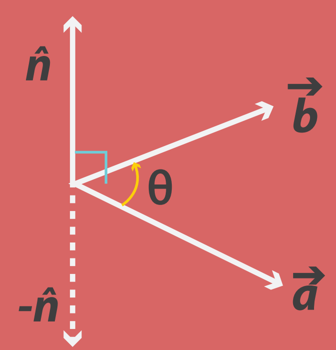
1. $\bar{a}.\bar{b}$ is a scalar quantity.
2. $\bar{a}.\bar{b} = \bar{0} \Leftrightarrow \bar{a} \perp \bar{b}$
3. $\bar{a}.\bar{b} = |\bar{a}|\,|\bar{b}| \Leftrightarrow \theta = 0^0$
4. $\bar{a}.\bar{b} = -\,|\bar{a}|\,|\bar{b}| \Leftrightarrow \theta = 180^0$
5. $\bar{i}.\bar{i} = \bar{j}.\bar{j} = \bar{k}.\bar{k} = 1$
6. $\bar{i}.\bar{j} = \bar{j}.\bar{k} = \bar{k}.\bar{i} = 0$
7. $\cos\theta = \dfrac{\bar{a}.\bar{b}}{|\bar{a}|\,|\bar{b}|}$
8. $\bar{a}.\bar{b} = \bar{b}.\bar{a}$
9. $\bar{a}.(\bar{b} + \bar{c}) = \bar{a}.\bar{b} + \bar{a}.\bar{c}$
10. if $\bar{a} = a_1\bar{i} + a_2\bar{j} + a_3\bar{k}$ and $\bar{b} = b_1\bar{i} + b_2\bar{j} + \bar{b}_3 k$ then $\bar{a}.\bar{b} = a_1b_1 + a_2b_2 + a_3b_3$
11. $(\lambda\bar{a}).\bar{b} = \lambda(\bar{a}.\bar{b}) = \bar{a}.(\lambda.\bar{b})$

Vector Product

For vectors $\bar{a}$ and $\bar{b}$ it is defined as

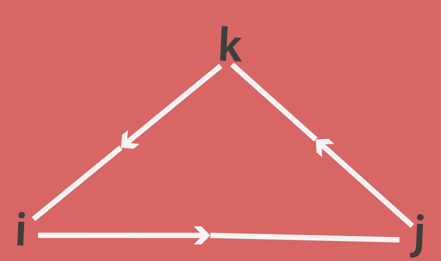
$$\bar{a} \times \bar{b} = |\bar{a}|\,|\bar{b}| \sin\theta\, \hat{n}$$

Where θ is an angle between $\bar{a}$ and $\bar{b}$, $0 \leq \theta \leq \pi$ and $\hat{n}$ is the unit vector perpendicular to both $\bar{a}$ and $\bar{b}$ such that $\bar{a}$, $\bar{b}$ and $\hat{n}$ form a right handed system



Properties Of Vector Product

1. $\bar{a} \times \bar{b}$ is a vector.
2. $\bar{a} \times \bar{b} = 0 \Leftrightarrow \bar{a} \parallel \bar{b}$
3. $\bar{i} \times \bar{i} = \bar{j} \times \bar{j} = \bar{k} \times \bar{k} = 0$
4. $\bar{i} \times \bar{j} = \bar{j} \times \bar{k} = \bar{i}, \bar{k} \times \bar{i} = \bar{j}$



5. $\sin\theta = \dfrac{|\bar{a} \times \bar{b}|}{|\bar{a}|.|\bar{b}|}$
6. $\bar{a} \times \bar{b} = -\bar{b} \times \bar{a}$
7. $\bar{a} \times (\bar{b} + \bar{c}) = \bar{a} \times \bar{b} + \bar{a} \times \bar{c}$
8. $\lambda(\bar{a} \times \bar{b}) = (\lambda\bar{a}) \times \bar{b} = \bar{a} \times (\lambda\bar{b})$
9. if $\bar{a} = a_1\bar{i} + a_2\bar{i} + a_3\bar{k}$ and $\bar{b} = b_1\bar{i} + b_2\bar{j} + b_3\bar{k}$ then

$$\bar{a} \times \bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$