Motion in a Plane - Part I

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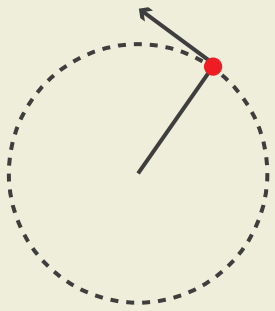
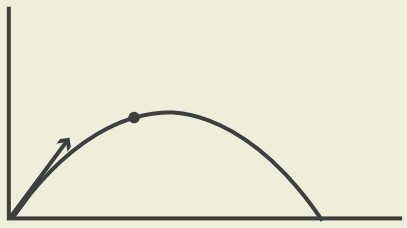
Motion in a plane

Examples of motion in two dimensions.



Circular motion



Projectile motion

Equations of motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

v = final velocity of the particle
u = initial velocity of the particle
s = displacement of the particle
a = acceleration of the particle
t = the time interval in which the particle is in consideration

Equations of motion in a plane

Apply equations of motion in a straight line separately in both directions, X and Y.

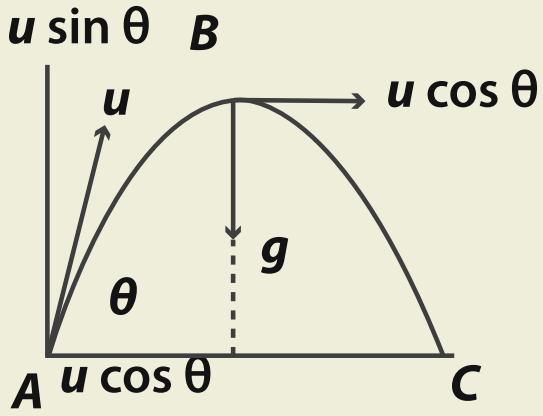
$$v_x = u_x + a_x t \qquad v_y = u_y + a_y t$$

$$s_x = u_x t + \frac{1}{2}a_x t^2 \qquad s_y = u_y t + \frac{1}{2}a_y t^2$$

$$v^2_x = u^2_x + 2a_y s \qquad v^2_y = u^2_y + 2a_y s$$

Projectile motion

- Projectile refers to an object that is in flight along the horizontal and vertical direction simultaneously.
- Acceleration acts only in the vertical direction due to acceleration due to gravity (g).
- No acceleration in the horizontal direction.
- Projectile motion is always in the form of parabola.



$$y = ax + bx^2$$

Formulas for projectile motion

Components of velocity at time t	$u_x = u\cos\theta$ $u_y = u\sin\theta - gt$
Position at time t	$x = (u\cos\theta)t$ $y = (u\sin\theta)t - 1/2\ gt^2$
Equation of path of projectile motion	$y = (\tan\theta)x - gx^2/2(u\cos\theta)^2$
Time of maximum height	$t_m = u\sin\theta/g$
Time of flight	$2t_m = 2(u\sin\theta/g)$
Maximum height of projectile	$h_m = (u\sin\theta)^2/2g$
Horizontal range of projectile	$R = u^2\sin 2\theta/g$
Maximum horizontal range ($\theta_0 = 45°$)	$R_m = u^2/g$

Motion in a Plane - Part II

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Velocity

Magnitude of the velocity vector is given by

$$|v| = v = \sqrt{v_x^2 + v_y^2}$$

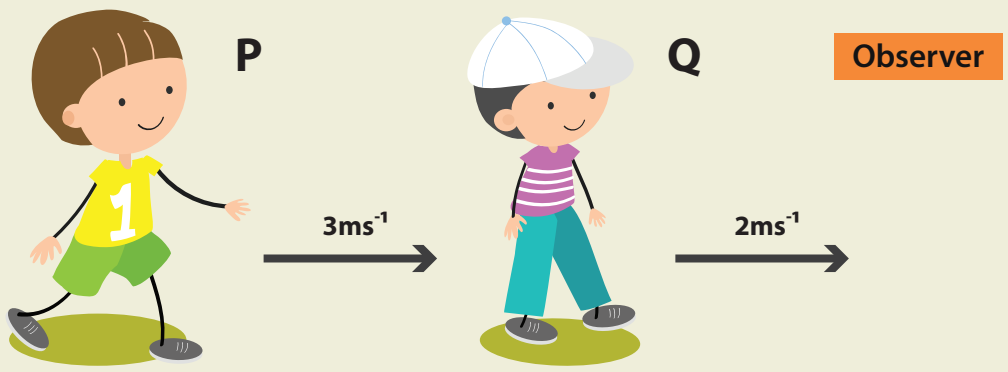
Acceleration

Rate of change of velocity of an object with respevct to time.

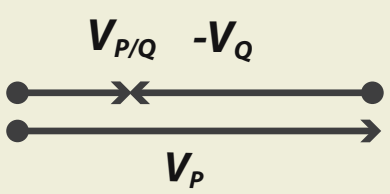
$$a_x = \frac{d}{dx} v_x \qquad a_y = \frac{d}{dx} v_y$$

Relative motion velocity

- Velocity of an object relative to some other object that might be stationary or moving with either same or different velocity.



- To the man Q, girl P appears to move at a speed of 1 m/s towards Q.
- To the girl P, man Q appears to move at a speed of 1 m/s towards P.
- Relative velocity equation, $V_p = V_{P/Q} + V_Q$
- Velocity diagram, $V_{P/Q} = V_P + (-V_Q)$



Relative velocity in 2 dimensions

$$V_{ab} = V_a - V_b$$
$$V_{ba} = V_b - V_a$$
$$V_{ba} = -V_{ab}$$
$$|V_{ab}| = |V_{ba}|$$

V_a , V_b = Velocity of object A and B with respect to a comman frame of reference.
V_{ab} = Velocity of a with respect to b.
V_{ba} = Velocity of b with respect to a.

- When two objects seem to be statiolnary for one another, in that case.

$$V_b = V_a$$
$$V_{ba} = V_{ab} = 0$$

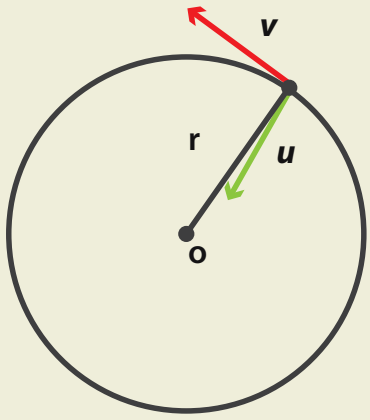
- If $V_b > 0$ and $V_a > 0$ or $V_b > 0$ and $V_a > 0$, and $V_a > V_b$ then Object A appears faster to B.

- The magnitude of V_{ba} and V_{ab} will be higher than the magnitude of V_a and V_b if V_a and V_b are of opposite sign. In this case, both objects will appear moving faster to one another.

Motion in a Plane - Part III

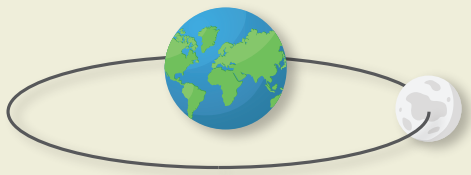
Circular motion

- Circular motion is the movement of an object in a circular path.



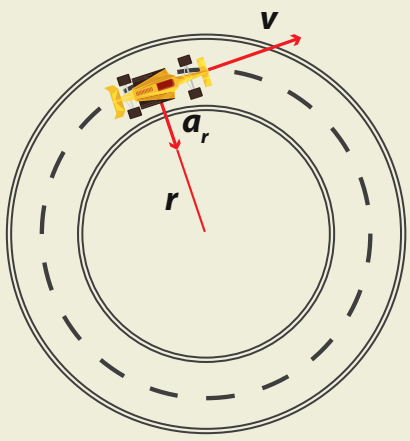
Uniform Circular motion

- Uniform circular motion can be described as the motion of an object in a circle at a constant speed.



Non-Uniform Circular motion

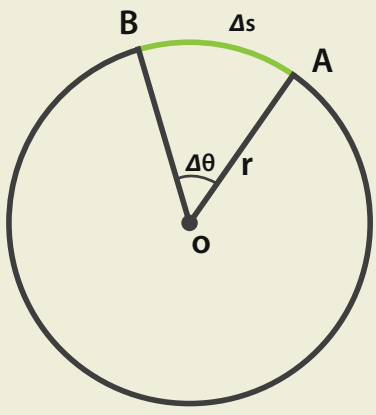
- Non-uniform circular motion can be described as the motion of an object in a circle where the speed is not constant.



Variables in Circular motion

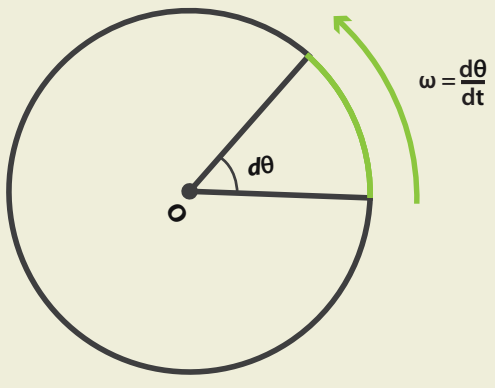
Angular Displacement

The angle which is subtended by the position vector at the centre of the circular path.
Unit: Radian (Rad)
Angular Displacement, $\theta = dS/r$
where, Δs = Linear displacement
r = Radius



Angular Velocity

Rate of change of angular displacement.
Unit: Rad/s
Angular velocity, $\omega = d\theta/dt$
Linear velocity, $v = r\omega$



Angular Acceleration

Rate of change of angular velocity.
Unit: rad/s^2

Angular Acceleration, $\alpha = \Delta\omega/\Delta t$
Linear acceleration, $a_t = r\alpha$

Centripetal Acceleration

Acceleration that acts on a body in circular motion whose direction is towards the centre of the circvle.
Centripetal Acceleration, $a_c = v^2/r$