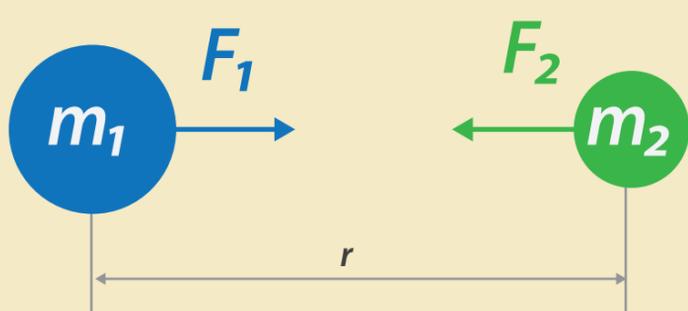


GRAVITATION

Newton's Law of Gravitation

Gravitational force (F) between two bodies is directly proportional to the product of masses and inversely proportional to the square of the distance between them.

$$\vec{F} = -\frac{Gm_1m_2}{r^2} \hat{r}$$



Acceleration due to Gravity

- For a body falling freely under gravity, the acceleration in the body is called the acceleration due to gravity.
- Relationship between g and G

$$g = \frac{GM_e}{R_e^2} = \frac{4}{3} \pi G R_e \rho$$

Where G = gravitational constant
 ρ = density of earth

M_e and R_e be the mass and radius of earth

Properties of Gravitational Force

- It is always attractive.
- It is independent of the medium.
- It is conservative and central force.
- It holds good over a wide range of distance.

Variation of g

Due to altitude (h)

The value of g goes on decreasing with height.

$$g' = g(1 - 2h/R)$$

Due to rotation of earth

Due to rotation of earth

$$g_\lambda = g - R_e \omega^2 \cos^2 \lambda$$

At equator, $\lambda = 0^\circ$

$$g_{\lambda_{min}} = g - R_e \omega^2$$

At poles, $\lambda = 90^\circ$

$$g_{\lambda_{max}} = g_p = g$$

Due to depth (d)

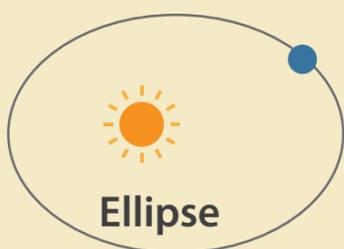
The value of g decreases with depth.

$$g' = g(1 - h/R)$$

Kepler's Laws of Planetary Motion

Law of Orbits

Every planet revolves around the sun in an elliptical orbit and the sun is situated at one of its foci.

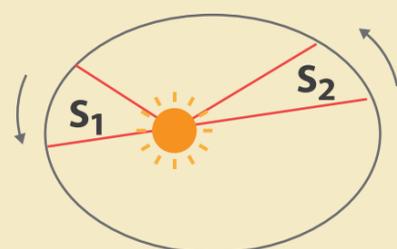


Ellipse

Law of Areas

The areal velocity of the planet around the sun is constant.

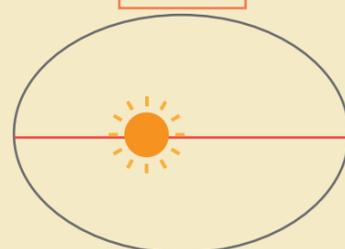
$$\text{i.e., } \frac{dA}{dt} = \text{a constant}$$



Law of Periods

The square of the time period of revolution of a planet is directly proportional to the cube of semi major axis of the elliptical orbit.

$$T^2 \propto a^3$$



Gravitational Potential Energy

Work done in bringing the given body from infinity to a point in the gravitational field.

$$U = \frac{-GMm}{r}$$

Gravitational Potential

Work done in bringing a unit mass from infinity to a point in the gravitational field.

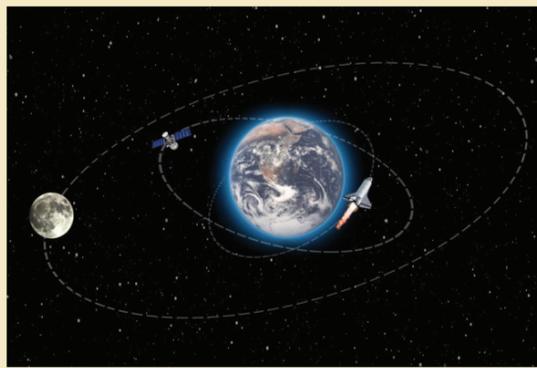
$$V = \frac{-GM}{r}$$

Escape Velocity

The minimum speed of projection of a body from surface of earth so that it just crosses the gravitational field of earth.

$$v_e = \sqrt{\frac{2GM}{R}}$$

Earth's Satellite



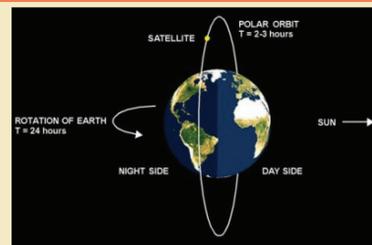
Geostationary Satellite

- Time period: 24 hours
- Same angular speed in same direction with earth
- Height: 36000 km.
- Uses: GPS, satellite communication (TV)



Polar Satellite

- Time period: 100 min
- Revolves in polar orbit around the earth.
- Height: 500-800 km.
- Uses: Weather forecasting, military spying



Orbital Velocity

- The minimum speed required to put the satellite into a given orbit.

$$v_o = R_e \sqrt{\frac{g}{R_e + h}}$$

- For satellite orbiting close to the earth's surface

$$v_o = \sqrt{gR_e}$$

Energy of a Satellite

- Kinetic energy $K = \frac{GM_e m}{2(R_e + h)}$
- Potential energy $U = \frac{-GM_e m}{R_e + h}$
- Total energy $E = K + U = \frac{-GM_e m}{2(R_e + h)}$

Time Period of a Satellite

For satellite orbiting close to the earth's surface

$$T = \frac{2\pi}{R_e} \sqrt{\frac{(R_e + h)^3}{g}}$$

$$T = 2\pi \sqrt{\frac{R_e}{g}} = 84.6 \text{ min}$$